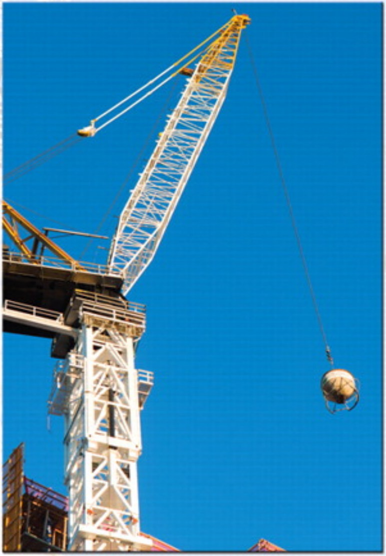
Engineering Mechanics

DYNAMICS

INSTRUCTOR SOLUTIONS MANUAL

Fourteenth Edition



R. C. Hibbeler

12–1.

Starting from rest, a particle moving in a straight line has an acceleration of $a = (2t - 6)\ \text{m/s}^2$, where t is in seconds. What is the particle's velocity when $t = 6$ s, and what is its position when $t = 11$ s?

SOLUTION

$a = 2t - 6$

$dv = a\,dt$

$$\int_0^v dv = \int_0^t (2t - 6)\,dt$$

$v = t^2 - 6t$

$ds = v\,dt$

$$\int_0^s ds = \int_0^t (t^2 - 6t)\,dt$$

$$s = \frac{t^3}{3} - 3t^2$$

When $t = 6\ s$,

$v = 0$ **Ans.**

When $t = 11\ s$,

$s = 80.7$ m **Ans.**

Ans:
$s = 80.7$ m

12–2.

If a particle has an initial velocity of $v_0 = 12\text{ ft/s}$ to the right, at $s_0 = 0$, determine its position when $t = 10\text{ s}$, if $a = 2\text{ ft/s}^2$ to the left.

SOLUTION

$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$= 0 + 12(10) + \frac{1}{2}(-2)(10)^2$$

$$= 20\text{ ft} \qquad \textbf{Ans.}$$

Ans:
$s = 20\text{ ft}$

12–3.

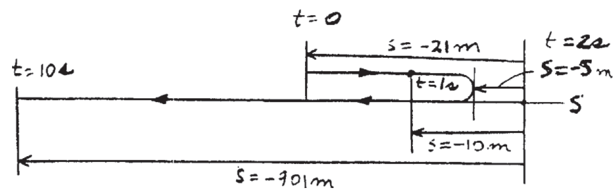
A particle travels along a straight line with a velocity $v = (12 - 3t^2)$ m/s, where t is in seconds. When $t = 1$ s, the particle is located 10 m to the left of the origin. Determine the acceleration when $t = 4$ s, the displacement from $t = 0$ to $t = 10$ s, and the distance the particle travels during this time period.

SOLUTION

$v = 12 - 3t^2$ **(1)**

$a = \dfrac{dv}{dt} = -6t\big|_{t=4} = -24 \text{ m/s}^2$ **Ans.**

$\int_{-10}^{s} ds = \int_{1}^{t} v\, dt = \int_{1}^{t} (12 - 3t^2)dt$



$s + 10 = 12t - t^3 - 11$

$s = 12t - t^3 - 21$

$s\big|_{t=0} = -21$

$s\big|_{t=10} = -901$

$\Delta s = -901 - (-21) = -880 \text{ m}$ **Ans.**

From Eq. (1):

$v = 0$ when $t = 2s$

$s\big|_{t=2} = 12(2) - (2)^3 - 21 = -5$

$s_T = (21 - 5) + (901 - 5) = 912 \text{ m}$ **Ans.**

Ans:
$a = -24 \text{ m/s}^2$
$\Delta s = -880 \text{ m}$
$s_T = 912 \text{ m}$

***12–4.**

A particle travels along a straight line with a constant acceleration. When $s = 4$ ft, $v = 3$ ft/s and when $s = 10$ ft, $v = 8$ ft/s. Determine the velocity as a function of position.

SOLUTION

Velocity: To determine the constant acceleration a_c, set $s_0 = 4$ ft, $v_0 = 3$ ft/s, $s = 10$ ft and $v = 8$ ft/s and apply Eq. 12–6.

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$8^2 = 3^2 + 2a_c\,(10 - 4)$$

$$a_c = 4.583 \text{ ft/s}^2$$

Using the result $a_c = 4.583$ ft/s^2, the velocity function can be obtained by applying Eq. 12–6.

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$v^2 = 3^2 + 2(4.583)\,(s - 4)$$

$$v = \left(\sqrt{9.17s - 27.7}\right) \text{ ft/s} \qquad \textbf{Ans.}$$

Ans:
$v = \left(\sqrt{9.17s - 27.7}\right)$ ft/s

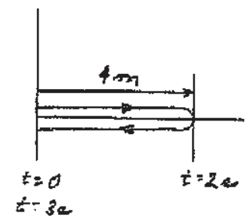
12–5.

The velocity of a particle traveling in a straight line is given by $v = (6t - 3t^2)$ m/s, where t is in seconds. If $s = 0$ when $t = 0$, determine the particle's deceleration and position when $t = 3$ s. How far has the particle traveled during the 3-s time interval, and what is its average speed?

SOLUTION

$v = 6t - 3t^2$

$a = \dfrac{dv}{dt} = 6 - 6t$



At $t = 3$ s

$a = -12 \text{ m/s}^2$ **Ans.**

$ds = v\,dt$

$$\int_0^s ds = \int_0^t (6t - 3t^2)dt$$

$s = 3t^2 - t^3$

At $t = 3$ s

$s = 0$ **Ans.**

Since $v = 0 = 6t - 3t^2$, when $t = 0$ and $t = 2$ s.

when $t = 2$ s, $s = 3(2)^2 - (2)^3 = 4$ m

$s_T = 4 + 4 = 8$ m **Ans.**

$(v_{sp})_{avg} = \dfrac{s_T}{t} = \dfrac{8}{3} = 2.67$ m/s **Ans.**

Ans:
$s_T = 8$ m
$v_{avg} = 2.67$ m/s

12–6.

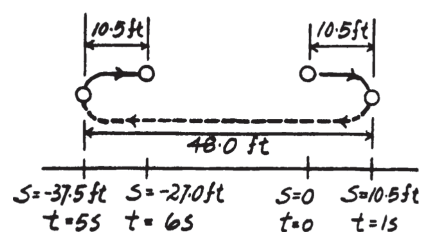
The position of a particle along a straight line is given by $s = (1.5t^3 - 13.5t^2 + 22.5t)$ ft, where t is in seconds. Determine the position of the particle when $t = 6$ s and the total distance it travels during the 6-s time interval. *Hint:* Plot the path to determine the total distance traveled.

SOLUTION

Position: The position of the particle when $t = 6$ s is

$$s|_{t=6s} = 1.5(6^3) - 13.5(6^2) + 22.5(6) = -27.0 \text{ ft}$$ **Ans.**

Total DistanceTraveled: The velocity of the particle can be determined by applying Eq. 12–1.



$$v = \frac{ds}{dt} = 4.50t^2 - 27.0t + 22.5$$

The times when the particle stops are

$$4.50t^2 - 27.0t + 22.5 = 0$$

$$t = 1 \text{ s} \qquad \text{and} \qquad t = 5 \text{ s}$$

The position of the particle at $t = 0$ s, 1 s and 5 s are

$$s|_{t=0\text{ s}} = 1.5(0^3) - 13.5(0^2) + 22.5(0) = 0$$

$$s|_{t=1\text{ s}} = 1.5(1^3) - 13.5(1^2) + 22.5(1) = 10.5 \text{ ft}$$

$$s|_{t=5\text{ s}} = 1.5(5^3) - 13.5(5^2) + 22.5(5) = -37.5 \text{ ft}$$

From the particle's path, the total distance is

$$s_{tot} = 10.5 + 48.0 + 10.5 = 69.0 \text{ ft}$$ **Ans.**

Ans:
$s|_{t=6\text{ s}} = -27.0$ ft
$s_{tot} = 69.0$ ft

12–7.

A particle moves along a straight line such that its position is defined by $s = (t^2 - 6t + 5)$ m. Determine the average velocity, the average speed, and the acceleration of the particle when $t = 6$ s.

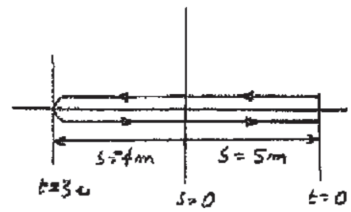
SOLUTION

$s = t^2 - 6t + 5$

$v = \frac{ds}{dt} = 2t - 6$

$a = \frac{dv}{dt} = 2$

$v = 0$ when $t = 3$



$s|_{t=0} = 5$

$s|_{t=3} = -4$

$s|_{t=6} = 5$

$v_{avg} = \frac{\Delta s}{\Delta t} = \frac{0}{6} = 0$ **Ans.**

$(v_{sp})_{avg} = \frac{s_T}{\Delta t} = \frac{9 + 9}{6} = 3 \text{ m/s}$ **Ans.**

$a|_{t=6} = 2 \text{ m/s}^2$ **Ans.**

Ans:
$v_{avg} = 0$
$(v_{sp})_{avg} = 3 \text{ m/s}$
$a|_{t=6\text{ s}} = 2 \text{ m/s}^2$

***12–8.**

A particle is moving along a straight line such that its position is defined by $s = (10t^2 + 20)$ mm, where t is in seconds. Determine (a) the displacement of the particle during the time interval from $t = 1$ s to $t = 5$ s, (b) the average velocity of the particle during this time interval, and (c) the acceleration when $t = 1$ s.

SOLUTION

$s = 10t^2 + 20$

(a) $s|_{1\,s} = 10(1)^2 + 20 = 30 \text{ mm}$

$s|_{5\,s} = 10(5)^2 + 20 = 270 \text{ mm}$

$\Delta s = 270 - 30 = 240 \text{ mm}$ **Ans.**

(b) $\Delta t = 5 - 1 = 4 \text{ s}$

$v_{avg} = \dfrac{\Delta s}{\Delta t} = \dfrac{240}{4} = 60 \text{ mm/s}$ **Ans.**

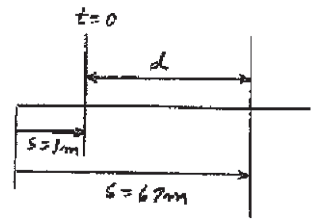
(c) $a = \dfrac{d^2 s}{dt^2} = 20 \text{ mm/s}^2$ (for all t) **Ans.**

Ans:
$\Delta s = 240$ mm
$v_{avg} = 60$ mm/s
$a = 20$ mm/s^2

12–9.

The acceleration of a particle as it moves along a straight line is given by $a = (2t - 1)\ \text{m/s}^2$, where t is in seconds. If $s = 1\ \text{m}$ and $v = 2\ \text{m/s}$ when $t = 0$, determine the particle's velocity and position when $t = 6\ \text{s}$. Also, determine the total distance the particle travels during this time period.

SOLUTION



$a = 2t - 1$

$dv = a\,dt$

$$\int_2^v dv = \int_0^t (2t - 1)dt$$

$v = t^2 - t + 2$

$dx = v\,dt$

$$\int_t^s ds = \int_0^t (t^2 - t + 2)dt$$

$$s = \frac{1}{3}t^3 - \frac{1}{2}t^2 + 2t + 1$$

When $t = 6\ \text{s}$

$v = 32\ \text{m/s}$ **Ans.**

$s = 67\ \text{m}$ **Ans.**

Since $v \neq 0$ for $0 \leq t \leq 6\ \text{s}$, then

$d = 67 - 1 = 66\ \text{m}$ **Ans.**

Ans:
$v = 32\ \text{m/s}$
$s = 67\ \text{m}$
$d = 66\ \text{m}$

12–10.

A particle moves along a straight line with an acceleration of $a = 5/(3s^{1/3} + s^{5/2})\ \text{m/s}^2$, where s is in meters. Determine the particle's velocity when $s = 2$ m, if it starts from rest when $s = 1$ m. Use a numerical method to evaluate the integral.

SOLUTION

$$a = \frac{5}{\left(3s^{\frac{1}{3}} + s^{\frac{5}{2}}\right)}$$

$$a\,ds = v\,dv$$

$$\int_1^2 \frac{5\,ds}{\left(3s^{\frac{1}{3}} + s^{\frac{5}{2}}\right)} = \int_0^v v\,dv$$

$$0.8351 = \frac{1}{2}v^2$$

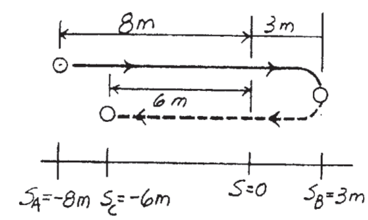
$v = 1.29$ m/s **Ans.**

Ans:
$v = 1.29$ m/s

12–11.

A particle travels along a straight-line path such that in 4 s it moves from an initial position $s_A = -8$ m to a position $s_B = +3$ m. Then in another 5 s it moves from s_B to $s_C = -6$ m. Determine the particle's average velocity and average speed during the 9-s time interval.

SOLUTION



Average Velocity: The displacement from A to C is $\Delta s = s_C - S_A = -6 - (-8) = 2$ m.

$$v_{avg} = \frac{\Delta s}{\Delta t} = \frac{2}{4+5} = 0.222 \text{ m/s} \qquad \textbf{Ans.}$$

Average Speed: The distances traveled from A to B and B to C are $s_{A \to B} = 8 + 3 = 11.0$ m and $s_{B \to C} = 3 + 6 = 9.00$ m, respectively. Then, the total distance traveled is $s_{Tot} = s_{A \to B} + s_{B \to C} = 11.0 + 9.00 = 20.0$ m.

$$(v_{sp})_{avg} = \frac{s_{Tot}}{\Delta t} = \frac{20.0}{4+5} = 2.22 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v_{avg} = 0.222$ m/s
$(v_{sp})_{avg} = 2.22$ m/s

***12–12.**

Traveling with an initial speed of 70 km/h, a car accelerates at 6000 km/h^2 along a straight road. How long will it take to reach a speed of 120 km/h? Also, through what distance does the car travel during this time?

SOLUTION

$v = v_1 + a_c t$

$120 = 70 + 6000(t)$

$t = 8.33(10^{-3})\text{ hr} = 30\text{ s}$ **Ans.**

$v^2 = v_1^2 + 2\,a_c(s - s_1)$

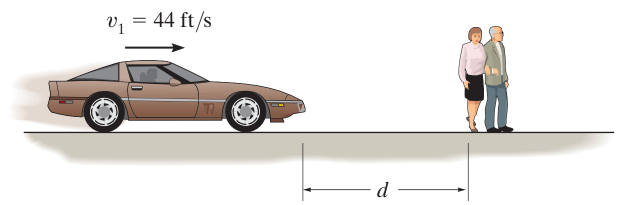
$(120)^2 = 70^2 + 2(6000)(s - 0)$

$s = 0.792\text{ km} = 792\text{ m}$ **Ans.**

Ans:
$t = 30\text{ s}$
$s = 792\text{ m}$

12–13.

Tests reveal that a normal driver takes about 0.75 s before he or she can *react* to a situation to avoid a collision. It takes about 3 s for a driver having 0.1% alcohol in his system to do the same. If such drivers are traveling on a straight road at 30 mph (44 ft/s) and their cars can decelerate at 2 ft/s^2, determine the shortest stopping distance d for each from the moment they see the pedestrians. *Moral*: If you must drink, please don't drive!



SOLUTION

Stopping Distance: For normal driver, the car moves a distance of $d' = vt = 44(0.75) = 33.0$ ft before he or she reacts and decelerates the car. The stopping distance can be obtained using Eq. 12–6 with $s_0 = d' = 33.0$ ft and $v = 0$.

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0^2 = 44^2 + 2(-2)(d - 33.0)$$

$$d = 517 \text{ ft} \qquad \textbf{Ans.}$$

For a drunk driver, the car moves a distance of $d' = vt = 44(3) = 132$ ft before he or she reacts and decelerates the car. The stopping distance can be obtained using Eq. 12–6 with $s_0 = d' = 132$ ft and $v = 0$.

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0^2 = 44^2 + 2(-2)(d - 132)$$

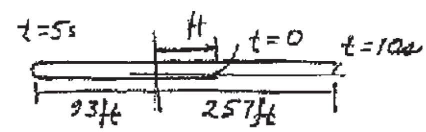
$$d = 616 \text{ ft} \qquad \textbf{Ans.}$$

Ans:
Normal: $d = 517$ ft
drunk: $d = 616$ ft

12–14.

The position of a particle along a straight-line path is defined by $s = (t^3 - 6t^2 - 15t + 7)$ ft, where t is in seconds. Determine the total distance traveled when $t = 10$ s. What are the particle's average velocity, average speed, and the instantaneous velocity and acceleration at this time?

SOLUTION



$s = t^3 - 6t^2 - 15t + 7$

$v = \dfrac{ds}{dt} = 3t^2 - 12t - 15$

When $t = 10$ s,

$v = 165$ ft/s **Ans.**

$a = \dfrac{dv}{dt} = 6t - 12$

When $t = 10$ s,

$a = 48$ ft/s^2 **Ans.**

When $v = 0$,

$0 = 3t^2 - 12t - 15$

The positive root is

$t = 5$ s

When $t = 0$, $\quad s = 7$ ft

When $t = 5$ s, $\quad s = -93$ ft

When $t = 10$ s, $\quad s = 257$ ft

Total distance traveled

$s_T = 7 + 93 + 93 + 257 = 450$ ft **Ans.**

$v_{avg} = \dfrac{\Delta s}{\Delta t} = \dfrac{257 - 7}{10 - 0} = 25.0$ ft/s **Ans.**

$(v_{sp})_{avg} = \dfrac{s_T}{\Delta t} = \dfrac{450}{10} = 45.0$ ft/s **Ans.**

Ans:
$v = 165$ ft/s
$a = 48$ ft/s^2
$s_T = 450$ ft
$v_{avg} = 25.0$ ft/s
$(v_{sp})_{avg} = 45.0$ ft/s

12–15.

A particle is moving with a velocity of v_0 when $s = 0$ and $t = 0$. If it is subjected to a deceleration of $a = -kv^3$, where k is a constant, determine its velocity and position as functions of time.

SOLUTION

$$a = \frac{d\nu}{dt} = -\mathrm{k}\nu^3$$

$$\int_{\nu 0}^{\nu} \nu^{-3}\, d\nu = \int_0^t -k\, dt$$

$$-\frac{1}{2}(\nu^{-2} - \nu_0^{-2}) = -kt$$

$$\nu = \left(2kt + \left(\frac{1}{\nu_0^2}\right)\right)^{-\frac{1}{2}}$$ **Ans.**

$$ds = \nu\, dt$$

$$\int_0^s ds = \int_0^t \frac{dt}{\left(2kt + \left(\frac{1}{\nu_0^2}\right)\right)^{\frac{1}{2}}}$$

$$s = \left.\frac{2\left(2kt + \left(\frac{1}{\nu_0^2}\right)\right)^{\frac{1}{2}}}{2k}\right|_0^t$$

$$s = \frac{1}{k}\left[\left(2kt + \left(\frac{1}{\nu_0^2}\right)\right)^{\frac{1}{2}} - \frac{1}{\nu_0}\right]$$ Ans.

Ans:

$$v = \left(2kt + \frac{1}{v_0^2}\right)^{-1/2}$$

$$s = \frac{1}{k}\left[\left(2kt + \frac{1}{v_0^2}\right)^{1/2} - \frac{1}{v_0}\right]$$

***12–16.**

A particle is moving along a straight line with an initial velocity of 6 m/s when it is subjected to a deceleration of $a = (-1.5v^{1/2})\ \text{m/s}^2$, where v is in m/s. Determine how far it travels before it stops. How much time does this take?

SOLUTION

Distance Traveled: The distance traveled by the particle can be determined by applying Eq. 12–3.

$$ds = \frac{v\,dv}{a}$$

$$\int_0^s ds = \int_{6\text{ m/s}}^{v} \frac{v}{-1.5v^{\frac{1}{2}}}\,dv$$

$$s = \int_{6\text{ m/s}}^{v} -0.6667\,v^{\frac{1}{2}}\,dv$$

$$= \left(-0.4444v^{\frac{3}{2}} + 6.532\right)\text{ m}$$

When $v = 0$, $\quad s = -0.4444\left(0^{\frac{3}{2}}\right) + 6.532 = 6.53\text{ m}$ **Ans.**

Time: The time required for the particle to stop can be determined by applying Eq. 12–2.

$$dt = \frac{dv}{a}$$

$$\int_0^t dt = -\int_{6\text{ m/s}}^{v} \frac{dv}{1.5v^{\frac{1}{2}}}$$

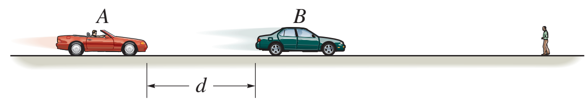
$$t = -1.333\left(v^{\frac{1}{2}}\right)\Big|_{6\text{ m/s}}^{v} = \left(3.266 - 1.333v^{\frac{1}{2}}\right)\text{s}$$

When $v = 0$, $\quad t = 3.266 - 1.333\left(0^{\frac{1}{2}}\right) = 3.27\text{ s}$ **Ans.**

Ans:
$s = 6.53$ m
$t = 3.27$ s

12–17.

Car B is traveling a distance d ahead of car A. Both cars are traveling at 60 ft/s when the driver of B suddenly applies the brakes, causing his car to decelerate at 12 ft/s^2. It takes the driver of car A 0.75 s to react (this is the normal reaction time for drivers). When he applies his brakes, he decelerates at 15 ft/s^2. Determine the minimum distance d be tween the cars so as to avoid a collision.



SOLUTION

For B:

$(\overset{+}{\rightarrow})\quad v = v_0 + a_c t$

$v_B = 60 - 12\,t$

$(\overset{+}{\rightarrow})\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$s_B = d + 60t - \frac{1}{2}(12)\,t^2 \qquad \textbf{(1)}$$

For A:

$(\overset{+}{\rightarrow})\quad v = v_0 + a_c t$

$v_A = 60 - 15(t - 0.75), \quad [t > 0.75]$

$(\overset{+}{\rightarrow})\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$s_A = 60(0.75) + 60(t - 0.75) - \frac{1}{2}(15)\,(t - 0.75)^2, \quad [t > 0.74] \qquad \textbf{(2)}$$

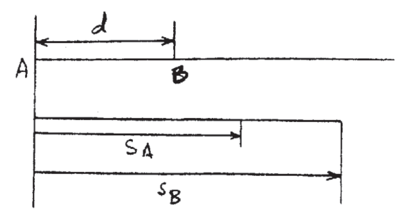
Require $v_A = v_B$ the moment of closest approach.

$60 - 12t = 60 - 15(t - 0.75)$

$t = 3.75$ s

Worst case without collision would occur when $s_A = s_B$.

At $t = 3.75$ s, from Eqs. (1) and (2):



$60(0.75) + 60(3.75 - 0.75) - 7.5(3.75 - 0.75)^2 = d + 60(3.75) - 6(3.75)^2$

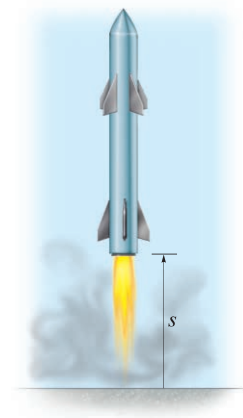
$157.5 = d + 140.625$

$d = 16.9$ ft **Ans.**

Ans:
$d = 16.9$ ft

12–18.

The acceleration of a rocket traveling upward is given by $a = (6 + 0.02s)\ \text{m/s}^2$, where s is in meters. Determine the time needed for the rocket to reach an altitude of $s = 100$ m. Initially, $v = 0$ and $s = 0$ when $t = 0$.



SOLUTION

$$a\,ds = v\,dv$$

$$\int_0^s (6 + 0.02\,s)\,ds = \int_0^v v\,dv$$

$$6\,s + 0.01\,s^2 = \frac{1}{2}v^2$$

$$v = \sqrt{12\,s + 0.02\,s^2}$$

$$ds = v\,dt$$

$$\int_0^{100} \frac{ds}{\sqrt{12\,s + 0.02\,s^2}} = \int_0^t dt$$

$$\frac{1}{\sqrt{0.02}}\ln\left[\sqrt{12s + 0.02s^2} + s\sqrt{0.02} + \frac{12}{2\sqrt{0.02}}\right]_0^{100} = t$$

$t = 5.62$ s **Ans.**

Ans:
$t = 5.62$ s

12–19.

A train starts from rest at station A and accelerates at $0.5\ \text{m/s}^2$ for 60 s. Afterwards it travels with a constant velocity for 15 min. It then decelerates at $1\ \text{m/s}^2$ until it is brought to rest at station B. Determine the distance between the stations.

SOLUTION

Kinematics: For stage (1) motion, $v_0 = 0$, $s_0 = 0$, $t = 60$ s, and $a_c = 0.5\ \text{m/s}^2$. Thus,

$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$s_1 = 0 + 0 + \frac{1}{2}(0.5)(60^2) = 900\ \text{m}$$

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t$$

$$v_1 = 0 + 0.5(60) = 30\ \text{m/s}$$

For stage (2) motion, $v_0 = 30\ \text{m/s}$, $s_0 = 900$ m, $a_c = 0$ and $t = 15(60) = 900$ s. Thus,

$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$s_2 = 900 + 30(900) + 0 = 27\,900\ \text{m}$$

For stage (3) motion, $v_0 = 30\ \text{m/s}$, $v = 0$, $s_0 = 27\,900$ m and $a_c = -1\ \text{m/s}^2$. Thus,

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t$$

$$0 = 30 + (-1)t$$

$$t = 30\ \text{s}$$

$$\overset{+}{\rightarrow} \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$s_3 = 27\,900 + 30(30) + \frac{1}{2}(-1)(30^2)$$

$$= 28\,350\ \text{m} = 28.4\ \text{km}$$ **Ans.**

Ans:
$s = 28.4$ km

***12–20.**

The velocity of a particle traveling along a straight line is $v = (3t^2 - 6t)$ ft/s, where t is in seconds. If $s = 4$ ft when $t = 0$, determine the position of the particle when $t = 4$ s. What is the total distance traveled during the time interval $t = 0$ to $t = 4$ s? Also, what is the acceleration when $t = 2$ s?

SOLUTION

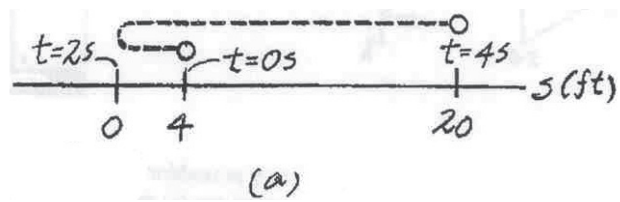
Position: The position of the particle can be determined by integrating the kinematic equation $ds = v\,dt$ using the initial condition $s = 4$ ft when $t = 0$ s. Thus,

$$(\overset{+}{\rightarrow}) \qquad ds = v\,dt$$

$$\int_{4\text{ ft}}^{s} ds = \int_0^t \left(3t^2 - 6t\right)dt$$

$$s\Big|_{4\text{ ft}}^{s} = \left(t^3 - 3t^2\right)\Big|_0^t$$

$$s = \left(t^3 - 3t^2 + 4\right)\text{ ft}$$



When $t = 4$ s,

$$s|_{4\text{ s}} = 4^3 - 3(4^2) + 4 = 20\text{ ft}$$ **Ans.**

The velocity of the particle changes direction at the instant when it is momentarily brought to rest. Thus,

$$v = 3t^2 - 6t = 0$$

$$t(3t - 6) = 0$$

$$t = 0 \text{ and } t = 2\text{ s}$$

The position of the particle at $t = 0$ and 2 s is

$$s|_{0\text{ s}} = 0 - 3\left(0^2\right) + 4 = 4\text{ ft}$$

$$s|_{2\text{ s}} = 2^3 - 3\left(2^2\right) + 4 = 0$$

Using the above result, the path of the particle shown in Fig. *a* is plotted. From this figure,

$$s_{\text{Tot}} = 4 + 20 = 24\text{ ft}$$ **Ans.**

Acceleration:

$$(\overset{+}{\rightarrow}) \qquad a = \frac{dv}{dt} = \frac{d}{dt}\left(3t^2 - 6t\right)$$

$$a = (6t - 6)\text{ ft/s}^2$$

When $t = 2$ s,

$$a|_{t=2\text{ s}} = 6(2) - 6 = 6\text{ ft/s}^2 \rightarrow$$ **Ans.**

Ans:
$s_{\text{Tot}} = 24$ ft
$a|_{t=2\text{ s}} = 6\text{ ft/s}^2 \rightarrow$

12–21.

A freight train travels at $v = 60(1 - e^{-t})$ ft/s, where t is the elapsed time in seconds. Determine the distance traveled in three seconds, and the acceleration at this time.

SOLUTION

$v = 60(1 - e^{-t})$

$$\int_0^s ds = \int v\,dt = \int_0^3 60(1 - e^{-t})dt$$

$s = 60(t + e^{-t})\Big|_0^3$

$s = 123$ ft **Ans.**

$$a = \frac{dv}{dt} = 60(e^{-t})$$

At $t = 3$ s

$a = 60e^{-3} = 2.99\text{ ft/s}^2$ **Ans.**

Ans:
$s = 123$ ft
$a = 2.99\text{ ft/s}^2$

12–22.

A sandbag is dropped from a balloon which is ascending vertically at a constant speed of 6 m/s. If the bag is released with the same upward velocity of 6 m/s when $t = 0$ and hits the ground when $t = 8$ s, determine the speed of the bag as it hits the ground and the altitude of the balloon at this instant.

SOLUTION

$(+\downarrow) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$h = 0 + (-6)(8) + \frac{1}{2}(9.81)(8)^2$$

$$= 265.92 \text{ m}$$

During $t = 8$ s, the balloon rises

$$h' = vt = 6(8) = 48 \text{ m}$$

Altitude $= h + h' = 265.92 + 48 = 314$ m **Ans.**

$(+\downarrow) \qquad v = v_0 + a_c t$

$v = -6 + 9.81(8) = 72.5$ m/s **Ans.**

Ans:
$h = 314$ m
$v = 72.5$ m/s

12–23.

A particle is moving along a straight line such that its acceleration is defined as $a = (-2v)\ \text{m/s}^2$, where v is in meters per second. If $v = 20\ \text{m/s}$ when $s = 0$ and $t = 0$, determine the particle's position, velocity, and acceleration as functions of time.

SOLUTION

$$a = -2v$$

$$\frac{dv}{dt} = -2v$$

$$\int_{20}^{v} \frac{dv}{v} = \int_{0}^{t} -2\,dt$$

$$\ln \frac{v}{20} = -2t$$

$$v = \left(20e^{-2t}\right)\ \text{m/s} \qquad \textbf{Ans.}$$

$$a = \frac{dv}{dt} = \left(-40e^{-2t}\right)\ \text{m/s}^2 \qquad \textbf{Ans.}$$

$$\int_{0}^{s} ds = v\,dt = \int_{0}^{t} (20e^{-2t})dt$$

$$s = -10e^{-2t}\big|_0^t = -10\left(e^{-2t} - 1\right)$$

$$s = 10\left(1 - e^{-2t}\right)\ \text{m} \qquad \textbf{Ans.}$$

Ans:
$v = \left(20e^{-2t}\right)\ \text{m/s}$
$a = \left(-40e^{-2t}\right)\ \text{m/s}^2$
$s = 10\left(1 - e^{-2t}\right)\ \text{m}$

***12–24.**

The acceleration of a particle traveling along a straight line is $a = \frac{1}{4}s^{1/2}$ m/s^2, where s is in meters. If $v = 0, s = 1$ m when $t = 0$, determine the particle's velocity at $s = 2$ m.

SOLUTION

Velocity:

$$(\overset{+}{\rightarrow}) \qquad v\,dv = a\,ds$$

$$\int_0^v v\,dv = \int_1^s \frac{1}{4}s^{1/2}ds$$

$$\frac{v^2}{2}\bigg|_0^v = \frac{1}{6}s^{3/2}\bigg|_1^s$$

$$v = \frac{1}{\sqrt{3}}(s^{3/2} - 1)^{1/2}\ \text{m/s}$$

When $s = 2$ m, $v = 0.781$ m/s. **Ans.**

Ans:
$v = 0.781$ m/s

12–25.

If the effects of atmospheric resistance are accounted for, a falling body has an acceleration defined by the equation $a = 9.81[1 - v^2(10^{-4})]\ \mathrm{m/s^2}$, where v is in m/s and the positive direction is downward. If the body is released from rest at a *very high altitude*, determine (a) the velocity when $t = 5$ s, and (b) the body's terminal or maximum attainable velocity (as $t \rightarrow \infty$).

SOLUTION

Velocity: The velocity of the particle can be related to the time by applying Eq. 12–2.

$$(+\downarrow) \qquad dt = \frac{dv}{a}$$

$$\int_0^t dt = \int_0^v \frac{dv}{9.81[1 - (0.01v)^2]}$$

$$t = \frac{1}{9.81}\left[\int_0^v \frac{dv}{2(1 + 0.01v)} + \int_0^v \frac{dv}{2(1 - 0.01v)}\right]$$

$$9.81t = 50\ln\left(\frac{1 + 0.01v}{1 - 0.01v}\right)$$

$$v = \frac{100(e^{0.1962t} - 1)}{e^{0.1962t} + 1} \qquad \textbf{(1)}$$

a) When $t = 5$ s, then, from Eq. (1)

$$v = \frac{100[e^{0.1962(5)} - 1]}{e^{0.1962(5)} + 1} = 45.5\ \mathrm{m/s} \qquad \textbf{Ans.}$$

b) If $t \rightarrow \infty$, $\dfrac{e^{0.1962t} - 1}{e^{0.1962t} + 1} \rightarrow 1$. Then, from Eq. (1)

$$v_{max} = 100\ \mathrm{m/s} \qquad \textbf{Ans.}$$

Ans:
(a) $v = 45.5\ \mathrm{m/s}$
(b) $v_{max} = 100\ \mathrm{m/s}$

12–26.

The acceleration of a particle along a straight line is defined by $a = (2t - 9)\ \text{m/s}^2$, where t is in seconds. At $t = 0$, $s = 1$ m and $v = 10$ m/s. When $t = 9$ s, determine (a) the particle's position, (b) the total distance traveled, and (c) the velocity.

SOLUTION

$a = 2t - 9$

$$\int_{10}^{v} dv = \int_{0}^{t} (2t - 9)\, dt$$

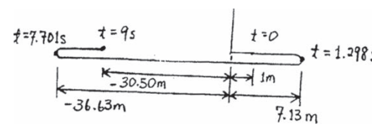
$v - 10 = t^2 - 9\,t$

$v = t^2 - 9\,t + 10$

$$\int_{1}^{s} ds = \int_{0}^{t} (t^2 - 9t + 10)\, dt$$

$s - 1 = \frac{1}{3}t^3 - 4.5\,t^2 + 10\,t$

$s = \frac{1}{3}t^3 - 4.5\,t^2 + 10\,t + 1$



Note when $v = t^2 - 9\,t + 10 = 0$:

$$t = 1.298 \text{ s and } t = 7.701 \text{ s}$$

When $t = 1.298$ s, $\quad s = 7.13$ m

When $t = 7.701$ s, $\quad s = -36.63$ m

When $t = 9$ s, $\quad s = -30.50$ m

(a) $\quad s = -30.5$ m **Ans.**

(b) $\quad s_{Tot} = (7.13 - 1) + 7.13 + 36.63 + (36.63 - 30.50)$

$\quad s_{Tot} = 56.0$ m **Ans.**

(c) $\quad v = 10$ m/s **Ans.**

Ans:
(a) $s = -30.5$ m
(b) $s_{\text{Tot}} = 56.0$ m
(c) $v = 10$ m/s

12–27.

When a particle falls through the air, its initial acceleration $a = g$ diminishes until it is zero, and thereafter it falls at a constant or terminal velocity v_f. If this variation of the acceleration can be expressed as $a = (g/v^2{}_f)(v^2{}_f - v^2)$, determine the time needed for the velocity to become $v = v_f/2$. Initially the particle falls from rest.

SOLUTION

$$\frac{dv}{dt} = a = \left(\frac{g}{v_f^2}\right)\left(v_f^2 - v^2\right)$$

$$\int_0^v \frac{dv}{v_f^2 - v^{2'}} = \frac{g}{v_f^2}\int_0^t dt$$

$$\frac{1}{2v_f}\ln\left(\frac{v_f + v}{v_f - v}\right)\Bigg|_0^v = \frac{g}{v_f^2}t$$

$$t = \frac{v_f}{2g}\ln\left(\frac{v_f + v}{v_f - v}\right)$$

$$t = \frac{v_f}{2g}\ln\left(\frac{v_f + v_f/2}{v_f - v_f/2}\right)$$

$$t = 0.549\left(\frac{v_f}{g}\right)$$ **Ans.**

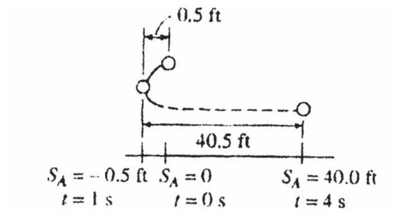
Ans:
$t = 0.549\left(\frac{v_f}{g}\right)$

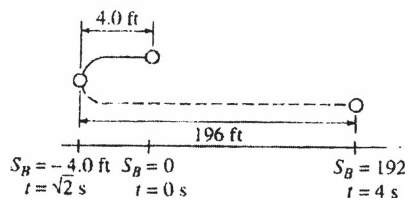
***12–28.**

Two particles A and B start from rest at the origin $s = 0$ and move along a straight line such that $a_A = (6t - 3)$ ft/s^2 and $a_B = (12t^2 - 8)$ ft/s^2, where t is in seconds. Determine the distance between them when $t = 4$ s and the total distance each has traveled in $t = 4$ s.

SOLUTION

Velocity: The velocity of particles A and B can be determined using Eq. 12-2.





$$dv_A = a_A dt$$

$$\int_0^{v_A} dv_A = \int_0^t (6t - 3)dt$$

$$v_A = 3t^2 - 3t$$

$$dv_B = a_B dt$$

$$\int_0^{v_B} dv_B = \int_0^t (12t^2 - 8)dt$$

$$v_B = 4t^3 - 8t$$

The times when particle A stops are

$3t^2 - 3t = 0 \qquad t = 0$ s and $= 1$ s

The times when particle B stops are

$4t^3 - 8t = 0 \quad t = 0$ s and $t = \sqrt{2}$ s

Position: The position of particles A and B can be determined using Eq. 12-1.

$$ds_A = v_A dt$$

$$\int_0^{s_A} ds_A = \int_0^t (3t^2 - 3t)dt$$

$$s_A = t^3 - \frac{3}{2}t^2$$

$$ds_B = v_B dt$$

$$\int_0^{s_B} ds_B = \int_0^t (4t^3 - 8t)dt$$

$$s_B = t^4 - 4t^2$$

The positions of particle A at $t = 1$ s and 4 s are

$$s_A|_{t=1\,s} = 1^3 - \frac{3}{2}(1^2) = -0.500 \text{ ft}$$

$$s_A|_{t=4\,s} = 4^3 - \frac{3}{2}(4^2) = 40.0 \text{ ft}$$

Particle A has traveled

$$d_A = 2(0.5) + 40.0 = 41.0 \text{ ft}$$ **Ans.**

The positions of particle B at $t = \sqrt{2}$ s and 4 s are

$$s_B|_{t=\sqrt{2}} = (\sqrt{2})^4 - 4(\sqrt{2})^2 = -4 \text{ ft}$$

$$s_B|_{t=4} = (4)^4 - 4(4)^2 = 192 \text{ ft}$$

Particle B has traveled

$$d_B = 2(4) + 192 = 200 \text{ ft}$$ **Ans.**

At $t = 4$ s the distance beween A and B is

$$\Delta s_{AB} = 192 - 40 = 152 \text{ ft}$$ **Ans.**

Ans:
$d_A = 41.0$ ft
$d_B = 200$ ft
$\Delta s_{AB} = 152$ ft

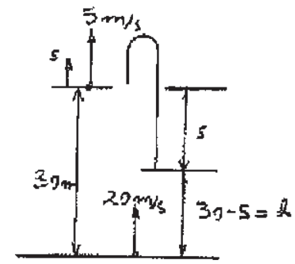
12–29.

A ball A is thrown vertically upward from the top of a 30-m-high building with an initial velocity of 5 m/s. At the same instant another ball B is thrown upward from the ground with an initial velocity of 20 m/s. Determine the height from the ground and the time at which they pass.

SOLUTION

Origin at roof:

Ball A:



$(+\uparrow)$ $$s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$-s = 0 + 5t - \frac{1}{2}(9.81)t^2$$

Ball B:

$(+\uparrow)$ $$s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$-s = -30 + 20t - \frac{1}{2}(9.81)t^2$$

Solving,

$t = 2\text{ s}$ **Ans.**

$s = 9.62\text{ m}$

Distance from ground,

$d = (30 - 9.62) = 20.4\text{ m}$ **Ans.**

Also, origin at ground,

$$s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$s_A = 30 + 5t + \frac{1}{2}(-9.81)t^2$$

$$s_B = 0 + 20t + \frac{1}{2}(-9.81)t^2$$

Require

$$s_A = s_B$$

$$30 + 5t + \frac{1}{2}(-9.81)t^2 = 20t + \frac{1}{2}(-9.81)t^2$$

$t = 2\text{ s}$ **Ans.**

$s_B = 20.4\text{ m}$ **Ans.**

Ans:
$h = 20.4\text{ m}$
$t = 2\text{ s}$

12–30.

A sphere is fired downwards into a medium with an initial speed of 27 m/s. If it experiences a deceleration of $a = (-6t)\ \text{m/s}^2$, where t is in seconds, determine the distance traveled before it stops.

SOLUTION

Velocity: $v_0 = 27$ m/s at $t_0 = 0$ s. Applying Eq. 12–2, we have

$$(+\downarrow) \qquad dv = a\,dt$$

$$\int_{27}^{v} dv = \int_0^t -6t\,dt$$

$$v = \left(27 - 3t^2\right)\ \text{m/s} \qquad \textbf{(1)}$$

At $v = 0$, from Eq. (1)

$$0 = 27 - 3t^2 \qquad t = 3.00\ \text{s}$$

Distance Traveled: $s_0 = 0$ m at $t_0 = 0$ s. Using the result $v = 27 - 3t^2$ and applying Eq. 12–1, we have

$$(+\downarrow) \qquad ds = v\,dt$$

$$\int_0^s ds = \int_0^t \left(27 - 3t^2\right) dt$$

$$s = \left(27t - t^3\right)\ \text{m} \qquad \textbf{(2)}$$

At $t = 3.00$ s, from Eq. (2)

$$s = 27(3.00) - 3.00^3 = 54.0\ \text{m} \qquad \textbf{Ans.}$$

Ans:
$s = 54.0$ m

12–31.

The velocity of a particle traveling along a straight line is $v = v_0 - ks$, where k is constant. If $s = 0$ when $t = 0$, determine the position and acceleration of the particle as a function of time.

SOLUTION

Position:

$$(\overset{+}{\rightarrow}) \qquad dt = \frac{ds}{v}$$

$$\int_0^t dt = \int_0^s \frac{ds}{v_0 - ks}$$

$$t\Big|_0^t = -\frac{1}{k}\ln(v_0 - ks)\Big|_0^s$$

$$t = \frac{1}{k}\ln\left(\frac{v_0}{v_0 - ks}\right)$$

$$e^{kt} = \frac{v_0}{v_0 - ks}$$

$$s = \frac{v_0}{k}\left(1 - e^{-kt}\right) \qquad \textbf{Ans.}$$

Velocity:

$$v = \frac{ds}{dt} = \frac{d}{dt}\left[\frac{v_0}{k}\left(1 - e^{-kt}\right)\right]$$

$$v = v_0 e^{-kt}$$

Acceleration:

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(v_0 e^{-kt}\right)$$

$$a = -kv_0 e^{-kt} \qquad \textbf{Ans.}$$

Ans:

$$s = \frac{v_0}{k}\left(1 - e^{-kt}\right)$$

$$a = -kv_0 e^{-kt}$$

***12–32.**

Ball A is thrown vertically upwards with a velocity of v_0. Ball B is thrown upwards from the same point with the same velocity t seconds later. Determine the elapsed time $t < 2v_0/g$ from the instant ball A is thrown to when the balls pass each other, and find the velocity of each ball at this instant.

SOLUTION

Kinematics: First, we will consider the motion of ball A with $(v_A)_0 = v_0$, $(s_A)_0 = 0$, $s_A = h, t_A = t'$, and $(a_c)_A = -g$.

$(+\uparrow) \quad s_A = (s_A)_0 + (v_A)_0 t_A + \frac{1}{2}(a_c)_A t_A^2$

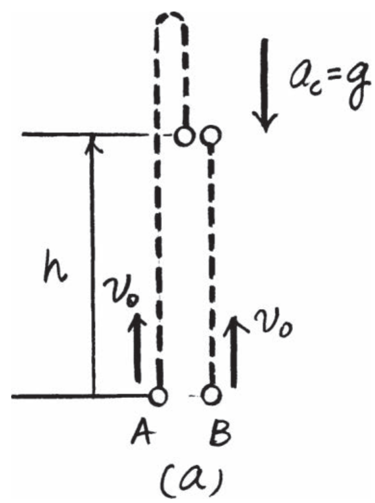
$$h = 0 + v_0 t' + \frac{1}{2}(-g)(t')^2$$

$$h = v_0 t' - \frac{g}{2}t'^2 \qquad \textbf{(1)}$$

$(+\uparrow) \quad v_A = (v_A)_0 + (a_c)_A t_A$

$$v_A = v_0 + (-g)(t')$$

$$v_A = v_0 - gt' \qquad \textbf{(2)}$$



The motion of ball B requires $(v_B)_0 = v_0$, $(s_B)_0 = 0$, $s_B = h$, $t_B = t' - t$, and $(a_c)_B = -g$.

$(+\uparrow) \quad s_B = (s_B)_0 + (v_B)_0 t_B + \frac{1}{2}(a_c)_B t_B^2$

$$h = 0 + v_0(t' - t) + \frac{1}{2}(-g)(t' - t)^2$$

$$h = v_0(t' - t) - \frac{g}{2}(t' - t)^2 \qquad \textbf{(3)}$$

$(+\uparrow) \quad v_B = (v_B)_0 + (a_c)_B t_B$

$$v_B = v_0 + (-g)(t' - t)$$

$$v_B = v_0 - g(t' - t) \qquad \textbf{(4)}$$

Solving Eqs. (1) and (3),

$$v_0 t' - \frac{g}{2}t'^2 = v_0(t' - t) - \frac{g}{2}(t' - t)^2$$

$$t' = \frac{2v_0 + gt}{2g} \qquad \textbf{Ans.}$$

Substituting this result into Eqs. (2) and (4),

$$v_A = v_0 - g\left(\frac{2v_0 + gt}{2g}\right)$$

$$= -\frac{1}{2}gt = \frac{1}{2}gt\downarrow \qquad \textbf{Ans.}$$

$$v_B = v_0 - g\left(\frac{2v_0 + gt}{2g} - t\right)$$

$$= \frac{1}{2}gt \uparrow \qquad \textbf{Ans.}$$

Ans:
$t' = \frac{2v_0 + gt}{2g}$
$v_A = \frac{1}{2}gt \downarrow$
$v_B = \frac{1}{2}gt \uparrow$

12–33.

As a body is projected to a high altitude above the earth's *surface*, the variation of the acceleration of gravity with respect to altitude y must be taken into account. Neglecting air resistance, this acceleration is determined from the formula $a = -g_0[R^2/(R + y)^2]$, where g_0 is the constant gravitational acceleration at sea level, R is the radius of the earth, and the positive direction is measured upward. If $g_0 = 9.81\text{ m/s}^2$ and $R = 6356$ km, determine the minimum initial velocity (escape velocity) at which a projectile should be shot vertically from the earth's surface so that it does not fall back to the earth. *Hint:* This requires that $v = 0$ as $y \rightarrow \infty$.

SOLUTION

$$v\,dv = a\,dy$$

$$\int_v^0 v\,dv = -g_0 R^2 \int_0^\infty \frac{dy}{(R + y)^2}$$

$$\frac{v^2}{2}\bigg|_v^0 = \frac{g_0 R^2}{R + y}\bigg|_0^\infty$$

$$v = \sqrt{2 g_0 R}$$

$$= \sqrt{2(9.81)(6356)(10)^3}$$

$$= 11167\text{ m/s} = 11.2\text{ km/s}$$ **Ans.**

Ans:
$v = 11.2\text{ km/s}$

12–34.

Accounting for the variation of gravitational acceleration a with respect to altitude y (see Prob. 12–36), derive an equation that relates the velocity of a freely falling particle to its altitude. Assume that the particle is released from rest at an altitude y_0 from the earth's surface. With what velocity does the particle strike the earth if it is released from rest at an altitude $y_0 = 500$ km? Use the numerical data in Prob. 12–36.

SOLUTION

From Prob. 12–36,

$$(+\uparrow) \qquad a = -g_0 \frac{R^2}{(R+y)^2}$$

Since $a\,dy = v\,dv$

then

$$-g_0 R^2 \int_{y_0}^{y} \frac{dy}{(R+y)^2} = \int_0^v v\,dv$$

$$g_0 R^2 \left[\frac{1}{R+y}\right]_{y_0}^{y} = \frac{v^2}{2}$$

$$g_0 R^2 \left[\frac{1}{R+y} - \frac{1}{R+y_0}\right] = \frac{v^2}{2}$$

Thus

$$v = -R\sqrt{\frac{2g_0(y_0 - y)}{(R+y)(R+y_0)}} \qquad \textbf{Ans.}$$

When $y_0 = 500$ km, $\quad y = 0$,

$$v = -6356(10^3)\sqrt{\frac{2(9.81)(500)(10^3)}{6356(6356 + 500)(10^6)}}$$

$$v = -3016 \text{ m/s} = 3.02 \text{ km/s} \downarrow \qquad \textbf{Ans.}$$

Ans:

$$v = -R\sqrt{\frac{2g_0(y_0 - y)}{(R+y)(R+y_0)}}$$

$v_{imp} = 3.02$ km/s

12–35.

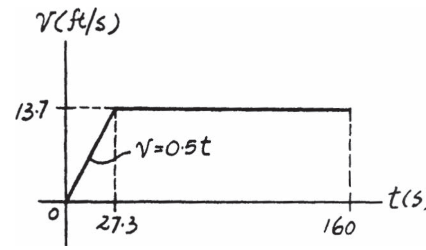
A freight train starts from rest and travels with a constant acceleration of 0.5 ft/s^2. After a time t' it maintains a constant speed so that when $t = 160$ s it has traveled 2000 ft. Determine the time t' and draw the v–t graph for the motion.

SOLUTION

Total Distance Traveled: The distance for part one of the motion can be related to time $t = t'$ by applying Eq. 12–5 with $s_0 = 0$ and $v_0 = 0$.

$$(\overset{+}{\rightarrow}) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_1 = 0 + 0 + \frac{1}{2}(0.5)(t')^2 = 0.25(t')^2$$



The velocity at time t can be obtained by applying Eq. 12–4 with $v_0 = 0$.

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t = 0 + 0.5t = 0.5t \qquad \textbf{(1)}$$

The time for the second stage of motion is $t_2 = 160 - t'$ and the train is traveling at a constant velocity of $v = 0.5t'$ (Eq. (1)). Thus, the distance for this part of motion is

$$(\overset{+}{\rightarrow}) \qquad s_2 = vt_2 = 0.5t'(160 - t') = 80t' - 0.5(t')^2$$

If the total distance traveled is $s_{Tot} = 2000$, then

$$s_{Tot} = s_1 + s_2$$

$$2000 = 0.25(t')^2 + 80t' - 0.5(t')^2$$

$$0.25(t')^2 - 80t' + 2000 = 0$$

Choose a root that is less than 160 s, then

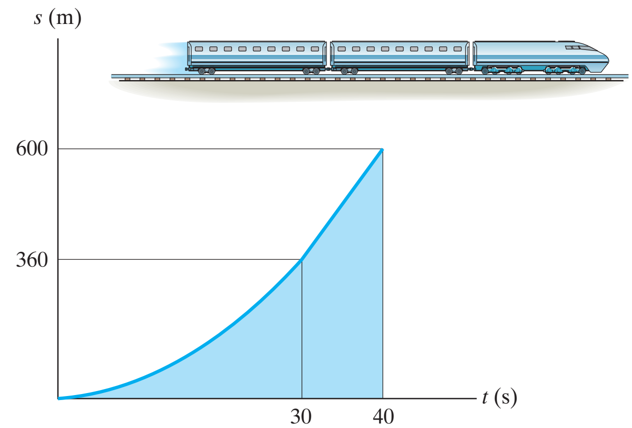
$$t' = 27.34 \text{ s} = 27.3 \text{ s} \qquad \textbf{Ans.}$$

v – t Graph: The equation for the velocity is given by Eq. (1). When $t = t' = 27.34$ s, $v = 0.5(27.34) = 13.7$ ft/s.

Ans:
$t' = 27.3$ s.
When $t = 27.3$ s, $v = 13.7$ ft/s.

***12–36.**

The s–t graph for a train has been experimentally determined. From the data, construct the v–t and a–t graphs for the motion; $0 \leq t \leq 40$ s. For $0 \leq t \leq 30$ s, the curve is $s = (0.4t^2)$ m, and then it becomes straight for $t \geq 30$ s.



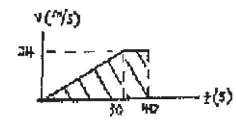
SOLUTION

$0 \leq t \leq 30$:

$$s = 0.4t^2$$

$$v = \frac{ds}{dt} = 0.8t$$

$$a = \frac{dv}{dt} = 0.8$$



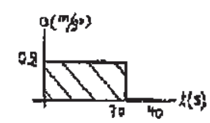
$30 \leq t \leq 40$:

$$s - 360 = \left(\frac{600 - 360}{40 - 30}\right)(t - 30)$$

$$s = 24(t - 30) + 360$$

$$v = \frac{ds}{dt} = 24$$

$$a = \frac{dv}{dt} = 0$$



Ans:

$$s = 0.4t^2$$

$$v = \frac{ds}{dt} = 0.8t$$

$$a = \frac{dv}{dt} = 0.8$$

$$s = 24(t - 30) + 360$$

$$v = \frac{ds}{dt} = 24$$

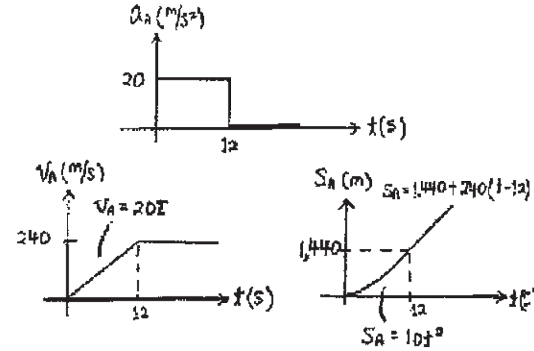
$$a = \frac{dv}{dt} = 0$$

12–37.

Two rockets start from rest at the same elevation. Rocket A accelerates vertically at 20 m/s^2 for 12 s and then maintains a constant speed. Rocket B accelerates at 15 m/s^2 until reaching a constant speed of 150 m/s. Construct the a–t, v–t, and s–t graphs for each rocket until $t = 20$ s. What is the distance between the rockets when $t = 20$ s?

SOLUTION

For rocket A



For $t < 12$ s

$$+\uparrow \; v_A = (v_A)_0 + a_A t$$

$$v_A = 0 + 20\,t$$

$$v_A = 20\,t$$

$$+\uparrow \; s_A = (s_A)_0 + (v_A)_0 t + \frac{1}{2} a_A t^2$$

$$s_A = 0 + 0 + \frac{1}{2}(20)\,t^2$$

$$s_A = 10\,t^2$$

When $t = 12$ s, $\quad v_A = 240$ m/s

$\qquad s_A = 1440$ m

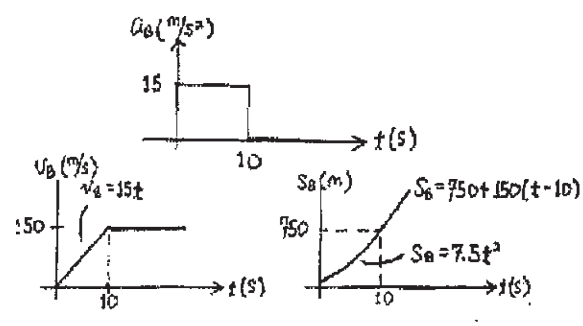
For $t > 12$ s

$v_A = 240$ m/s

$s_A = 1440 + 240(t - 12)$

For rocket B

For $t < 10$ s



$$+\uparrow \; v_B = (v_B)_0 + a_B t$$

$$v_B = 0 + 15\,t$$

$$v_B = 15\,t$$

$$+\uparrow \; s_B = (s_B)_0 + (v_B)_0 t + \frac{1}{2} a_B t^2$$

$$s_B = 0 + 0 + \frac{1}{2}(15)\,t^2$$

$$s_B = 7.5\,t^2$$

When $t = 10$ s, $\quad v_B = 150$ m/s

$\qquad s_B = 750$ m

For $t > 10$ s

$v_B = 150$ m/s

$s_B = 750 + 150(t - 10)$

When $t = 20$ s, $\quad s_A = 3360$ m, $\quad s_B = 2250$ m

$\Delta s = 1110$ m $= 1.11$ km **Ans.**

Ans:
$\Delta s = 1.11$ km

12–38.

A particle starts from $s = 0$ and travels along a straight line with a velocity $v = (t^2 - 4t + 3)$ m/s, where t is in seconds. Construct the $v-t$ and $a-t$ graphs for the time interval $0 \leq t \leq 4$ s.

SOLUTION

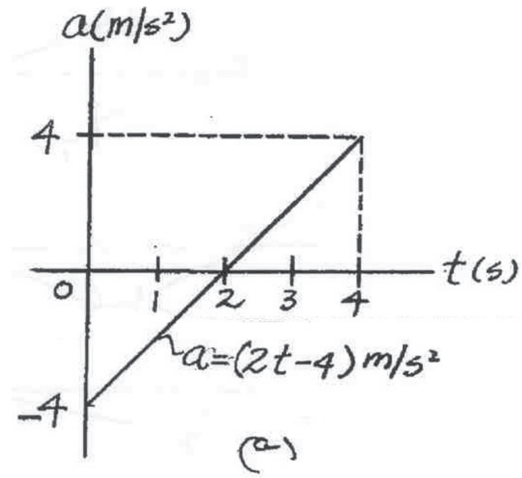
a–t Graph:

$$a = \frac{dv}{dt} = \frac{d}{dt}(t^2 - 4t + 3)$$
$$a = (2t - 4)\ \text{m/s}^2$$

Thus,

$$a|_{t=0} = 2(0) - 4 = -4\ \text{m/s}^2$$
$$a|_{t=2} = 0$$
$$a|_{t=4\,\text{s}} = 2(4) - 4 = 4\ \text{m/s}^2$$

The $a-t$ graph is shown in Fig. a.



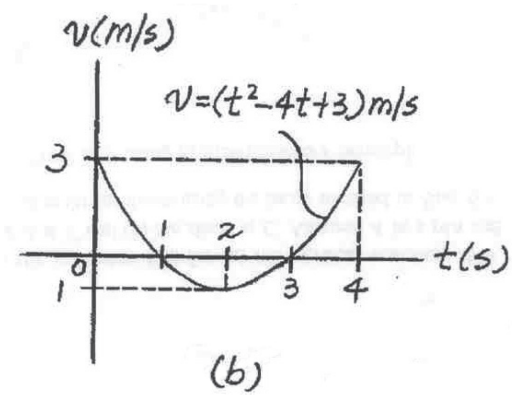
v–t Graph: The slope of the $v-t$ graph is zero when $a = \dfrac{dv}{dt} = 0$. Thus,

$$a = 2t - 4 = 0 \qquad t = 2\ \text{s}$$

The velocity of the particle at $t = 0$ s, 2 s, and 4 s are

$$v|_{t=0\,\text{s}} = 0^2 - 4(0) + 3 = 3\ \text{m/s}$$
$$v|_{t=2\,\text{s}} = 2^2 - 4(2) + 3 = -1\ \text{m/s}$$
$$v|_{t=4\,\text{s}} = 4^2 - 4(4) + 3 = 3\ \text{m/s}$$

The $v-t$ graph is shown in Fig. b.

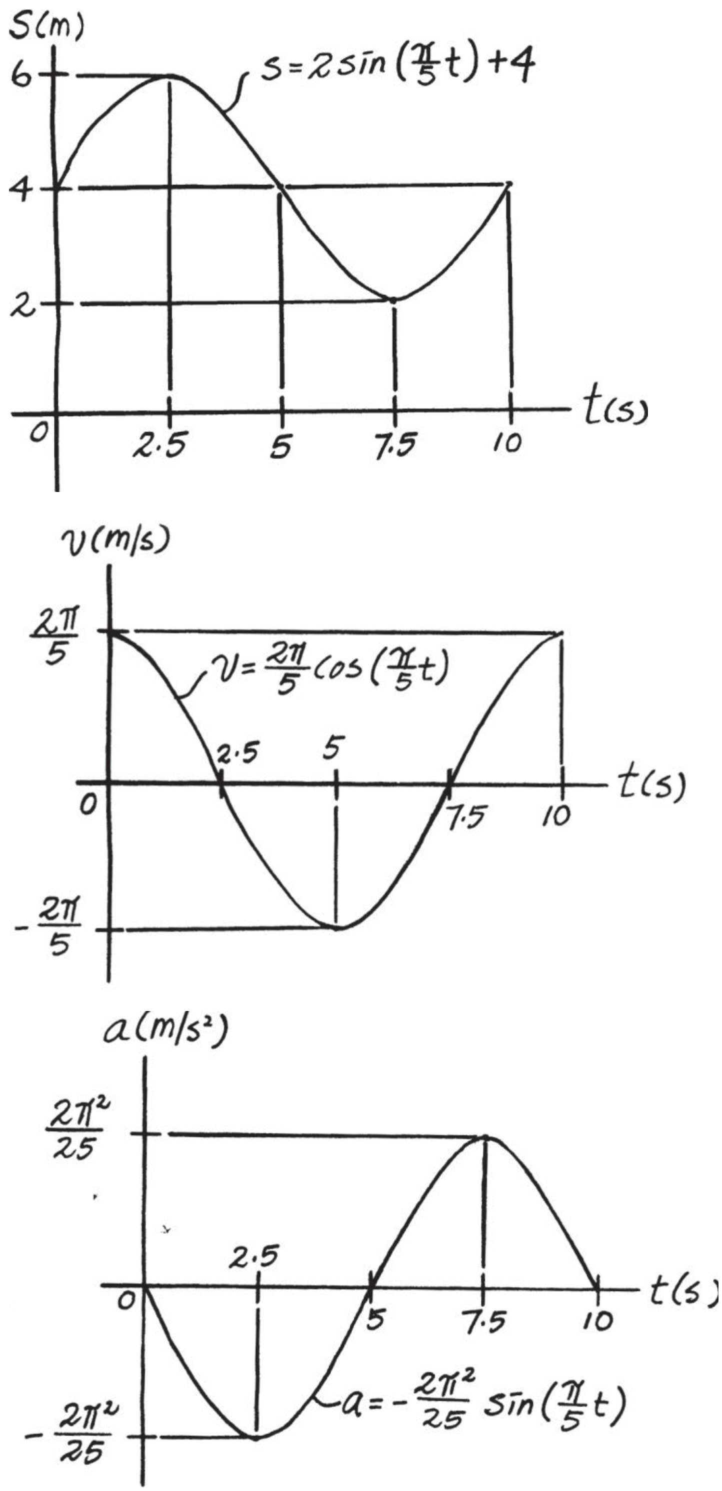


Ans:
$a|_{t=0} = -4\ \text{m/s}^2$
$a|_{t=2\,\text{s}} = 0$
$a|_{t=4\,\text{s}} = 4\ \text{m/s}^2$
$v|_{t=0} = 3\ \text{m/s}$
$v|_{t=2\,\text{s}} = -1\ \text{m/s}$
$v|_{t=4\,\text{s}} = 3\ \text{m/s}$

12–39.

If the position of a particle is defined by $s = [2\sin(\pi/5)t + 4]$ m, where t is in seconds, construct the $s-t$, $v-t$, and $a-t$ graphs for $0 \le t \le 10$ s.

SOLUTION



Ans:

$$s = 2\sin\left(\frac{\pi}{5}t\right) + 4$$

$$v = \frac{2\pi}{5}\cos\left(\frac{\pi}{5}t\right)$$

$$a = -\frac{2\pi^2}{25}\sin\left(\frac{\pi}{5}t\right)$$

***12–40.**

An airplane starts from rest, travels 5000 ft down a runway, and after uniform acceleration, takes off with a speed of 162 mi/h. It then climbs in a straight line with a uniform acceleration of 3 ft/s^2 until it reaches a constant speed of 220 mi/h. Draw the *s–t*, *v–t*, and *a–t* graphs that describe the motion.

SOLUTION

$v_1 = 0$

$v_2 = 162 \frac{\text{mi}}{\text{h}} \frac{(1\text{h})\ 5280\ \text{ft}}{(3600\ \text{s})(1\ \text{mi})} = 237.6\ \text{ft/s}$

$v_2^2 = v_1^2 + 2\,a_c(s_2 - s_1)$

$(237.6)^2 = 0^2 + 2(a_c)(5000 - 0)$

$a_c = 5.64538\ \text{ft/s}^2$

$v_2 = v_1 + a_c t$

$237.6 = 0 + 5.64538\,t$

$t = 42.09 = 42.1\ \text{s}$

$v_3 = 220 \frac{\text{mi}}{\text{h}} \frac{(1\text{h})\ 5280\ \text{ft}}{(3600\ \text{s})(1\ \text{mi})} = 322.67\ \text{ft/s}$

$v_3^2 = v_2^2 + 2a_c(s_3 - s_2)$

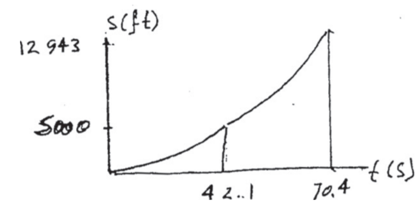
$(322.67)^2 = (237.6)^2 + 2(3)(s - 5000)$

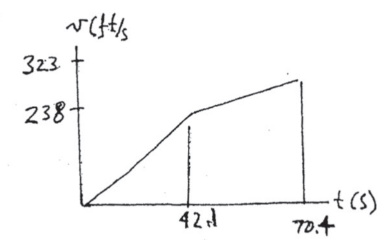
$s = 12\,943.34\ \text{ft}$

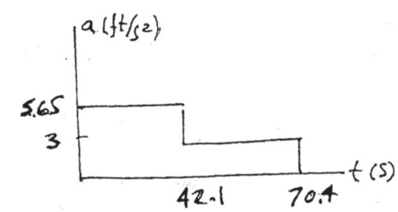
$v_3 = v_2 + a_c t$

$322.67 = 237.6 + 3\,t$

$t = 28.4\ \text{s}$



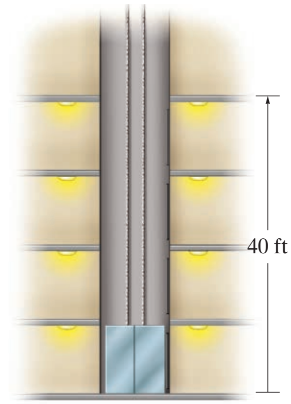




Ans:
$s = 12\,943.34\ \text{ft}$
$v_3 = v_2 + a_c t$
$t = 28.4\ \text{s}$

12–41.

The elevator starts from rest at the first floor of the building. It can accelerate at 5 ft/s^2 and then decelerate at 2 ft/s^2. Determine the shortest time it takes to reach a floor 40 ft above the ground. The elevator starts from rest and then stops. Draw the a–t, v–t, and s–t graphs for the motion.



SOLUTION

$+\uparrow\ v_2 = v_1 + a_c t_1$

$v_{max} = 0 + 5\,t_1$

$+\uparrow\ v_3 = v_2 + a_c\,t$

$0 = v_{max} - 2\,t_2$

Thus

$t_1 = 0.4\,t_2$

$+\uparrow\ s_2 = s_1 + v_1 t_1 + \frac{1}{2} a_c t_1^2$

$h = 0 + 0 + \frac{1}{2}(5)(t_1^2) = 2.5\,t_1^2$

$+\uparrow\ 40 - h = 0 + v_{max} t_2 - \frac{1}{2}(2)\,t_2^2$

$+\uparrow\ v^2 = v_1^2 + 2\,a_c(s - s_1)$

$v_{max}^2 = 0 + 2(5)(h - 0)$

$v_{max}^2 = 10h$

$0 = v_{max}^2 + 2(-2)(40 - h)$

$v_{max}^2 = 160 - 4h$

Thus,

$10\,h = 160 - 4h$

$h = 11.429$ ft

$v_{max} = 10.69$ ft/s

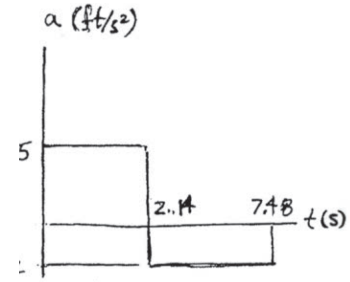
$t_1 = 2.138$ s

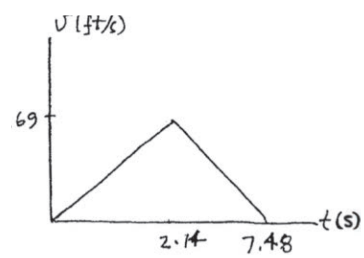
$t_2 = 5.345$ s

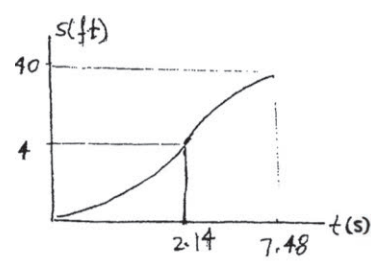
$t = t_1 + t_2 = 7.48$ s **Ans.**

When $t = 2.145$, $v = v_{max} = 10.7$ ft/s

and $h = 11.4$ ft.



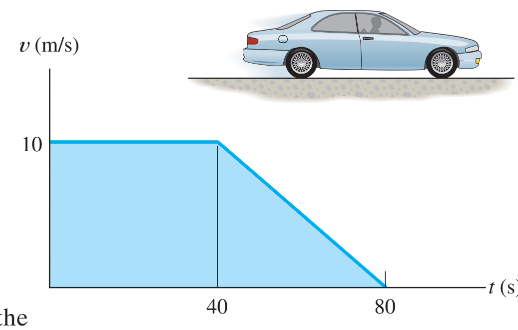




Ans:
$t = 7.48$ s. When $t = 2.14$ s,
$v = v_{max} = 10.7$ ft/s
$h = 11.4$ ft

12–42.

The velocity of a car is plotted as shown. Determine the total distance the car moves until it stops ($t = 80$ s). Construct the a–t graph.



SOLUTION

Distance Traveled: The total distance traveled can be obtained by computing the area under the $v - t$ graph.

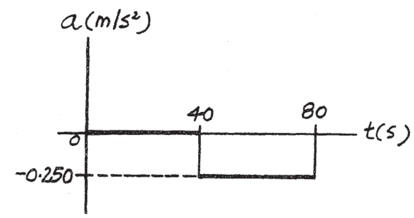
$$s = 10(40) + \frac{1}{2}(10)(80 - 40) = 600 \text{ m} \qquad \textbf{Ans.}$$

a – t Graph: The acceleration in terms of time t can be obtained by applying $a = \dfrac{dv}{dt}$. For time interval $0 \text{ s} \leq t < 40$ s,

$$a = \frac{dv}{dt} = 0$$

For time interval $40 \text{ s} < t \leq 80$ s, $\dfrac{v - 10}{t - 40} = \dfrac{0 - 10}{80 - 40}$, $v = \left(-\dfrac{1}{4}t + 20\right)$ m/s.

$$a = \frac{dv}{dt} = -\frac{1}{4} = -0.250 \text{ m/s}^2$$



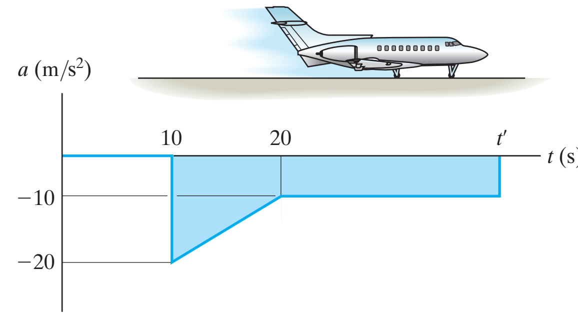
For $0 \leq t < 40$ s, $a = 0$.

For $40 \text{ s} < t \leq 80$, $a = -0.250 \text{ m/s}^2$.

Ans:
$s = 600$ m. For $0 \leq t < 40$ s, $a = 0$. For $40 \text{ s} < t \leq 80$ s, $a = -0.250 \text{ m/s}^2$

12–43.

The motion of a jet plane just after landing on a runway is described by the a–t graph. Determine the time t' when the jet plane stops. Construct the v–t and s–t graphs for the motion. Here $s = 0$, and $v = 300$ ft/s when $t = 0$.



SOLUTION

***v–t* Graph.** The v–t function can be determined by integrating $dv = a\,dt$. For $0 \le t < 10$ s, $a = 0$. Using the initial condition $v = 300$ ft/s at $t = 0$,

$$\int_{300\text{ ft/s}}^{v} dv = \int_0^t 0\,dt$$

$$v - 300 = 0$$

$$v = 300\text{ ft/s} \qquad \textbf{Ans.}$$

For $10\text{ s} < t < 20\text{ s}$, $\dfrac{a-(-20)}{t-10} = \dfrac{-10-(-20)}{20-10}$, $a = (t - 30)\text{ ft/s}^2$. Using the initial condition $v = 300$ ft/s at $t = 10$ s,

$$\int_{300\text{ ft/s}}^{v} dv = \int_{10\text{ s}}^{t} (t-30)\,dt$$

$$v - 300 = \left(\frac{1}{2}t^2 - 30t\right)\Big|_{10\text{ s}}^{t}$$

$$v = \left\{\frac{1}{2}t^2 - 30t + 550\right\}\text{ ft/s} \qquad \textbf{Ans.}$$

At $t = 20$ s,

$$v\Big|_{t=20\text{ s}} = \frac{1}{2}(20^2) - 30(20) + 550 = 150\text{ ft/s}$$

For $20\text{ s} < t < t'$, $a = -10$ ft/s. Using the initial condition $v = 150$ ft/s at $t = 20$ s,

$$\int_{150\text{ ft/s}}^{v} dv = \int_{20\text{ s}}^{t} -10\,dt$$

$$v - 150 = (-10t)\Big|_{20\text{ s}}^{t}$$

$$v = (-10t + 350)\text{ ft/s} \qquad \textbf{Ans.}$$

It is required that at $t = t'$, $v = 0$. Thus

$$0 = -10\,t' + 350$$

$$t' = 35\text{ s} \qquad \textbf{Ans.}$$

Using these results, the v–t graph shown in Fig. a can be plotted ***s-t* Graph**. The s–t function can be determined by integrating $ds = v\,dt$. For $0 \le t < 10$ s, the initial condition is $s = 0$ at $t = 0$.

$$\int_0^s ds = \int_0^t 300\,dt$$

$$s = \{300\,t\}\text{ ft} \qquad \textbf{Ans.}$$

At $= 10$ s,

$$s|_{t=10\text{ s}} = 300(10) = 3000\text{ ft}$$

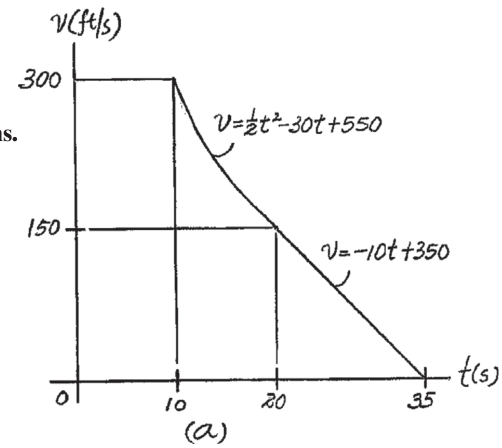
12–43. Continued

For 10 s < t < 20 s, the initial condition is $s = 3000$ ft at $t = 10$ s.

$$\int_{3000\text{ ft}}^{s} ds = \int_{10\text{ s}}^{t} \left(\frac{1}{2}t^2 - 30t + 550\right)dt$$

$$s - 3000 = \left(\frac{1}{6}t^3 - 15t^2 + 550t\right)\bigg|_{10\text{ s}}^{t}$$

$$s = \left\{\frac{1}{6}t^3 - 15t^2 + 550t - 1167\right\}\text{ ft}$$ **Ans.**



At $t = 20$ s,

$$s = \frac{1}{6}(20^3) - 15(20^2) + 550(20) - 1167 = 5167\text{ ft}$$

For 20 s < $t \leq 35$ s, the initial condition is $s = 5167$ ft at $t = 20$ s.

$$\int_{5167\text{ ft}}^{s} ds = \int_{20\text{ s}}^{t} (-10t + 350)\,dt$$

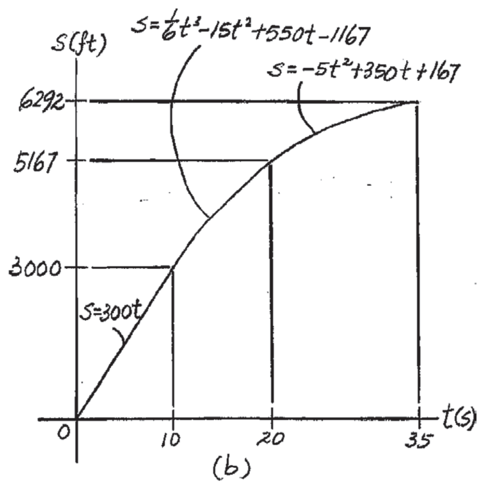
$$s - 5167 = (-5t^2 + 350t)\bigg|_{20\text{ s}}^{t}$$

$$s = \{-5t^2 + 350t + 167\}\text{ ft}$$ **Ans.**

At $t = 35$ s,

$$s\bigg|_{t = 35\text{ s}} = -5(35^2) + 350(35) + 167 = 6292\text{ ft}$$

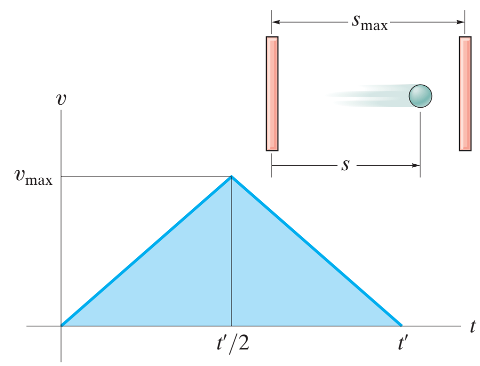
using these results, the *s-t* graph shown in Fig. *b* can be plotted.



Ans:
$t' = 35$ s
For $0 \leq t < 10$ s,
$s = \{300t\}$ ft
$v = 300$ ft/s
For 10 s < t < 20 s,
$s = \left\{\frac{1}{6}t^3 - 15t^2 + 550t - 1167\right\}$ ft
$v = \left\{\frac{1}{2}t^2 - 30t + 550\right\}$ ft/s
For 20 s < $t \leq 35$ s,
$s = \{-5t^2 + 350t + 167\}$ ft
$v = (-10t + 350)$ ft/s

***12–44.**

The v–t graph for a particle moving through an electric field from one plate to another has the shape shown in the figure. The acceleration and deceleration that occur are constant and both have a magnitude of 4 m/s^2. If the plates are spaced 200 mm apart, determine the maximum velocity v_{max} and the time t' for the particle to travel from one plate to the other. Also draw the s–t graph. When $t = t'/2$ the particle is at $s = 100$ mm.



SOLUTION

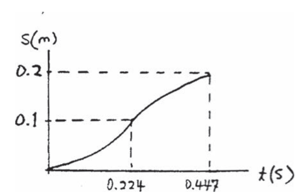
$a_c = 4\text{ m/s}^2$

$\frac{s}{2} = 100\text{ mm} = 0.1\text{ m}$

$v^2 = v_0^2 + 2\,a_c(s - s_0)$

$v_{max}^2 = 0 + 2(4)(0.1 - 0)$

$v_{max} = 0.89442\text{ m/s} \qquad = 0.894\text{ m/s}$ **Ans.**



$v = v_0 + a_c\,t'$

$0.89442 = 0 + 4(\frac{t'}{2})$

$t' = 0.44721\text{ s} \qquad = 0.447\text{ s}$ **Ans.**

$s = s_0 + v_0\,t + \frac{1}{2}\,a_c\,t^2$

$s = 0 + 0 + \frac{1}{2}\,(4)(t)^2$

$s = 2\,t^2$

When $t = \frac{0.44721}{2} = 0.2236 = 0.224\text{ s},$

$s = 0.1\text{ m}$

$\int_{0.894}^{v} ds = -\int_{0.2235}^{t} 4\,dt$

$v = -4\,t + 1.788$

$\int_{0.1}^{s} ds = \int_{0.2235}^{t} (-4t + 1.788)\,dt$

$s = -2\,t^2 + 1.788\,t - 0.2$

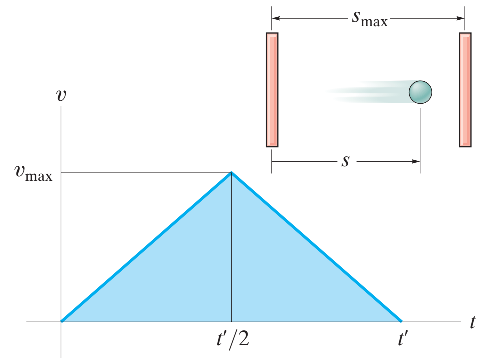
When $t = 0.447$ s,

$s = 0.2\text{ m}$

Ans:
$t' = 0.447\text{ s}$
$s = 0.2\text{ m}$

12–45.

The v–t graph for a particle moving through an electric field from one plate to another has the shape shown in the figure, where $t' = 0.2$ s and $v_{max} = 10$ m/s. Draw the s–t and a–t graphs for the particle. When $t = t'/2$ the particle is at $s = 0.5$ m.



SOLUTION

For $0 < t < 0.1\ s$,

$$v = 100\,t$$

$$a = \frac{dv}{dt} = 100$$

$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t 100\,t\,dt$$

$$s = 50\,t^2$$

When $t = 0.1$ s,

$$s = 0.5 \text{ m}$$

For $0.1\text{ s} < t < 0.2$ s,

$$v = -100\,t + 20$$

$$a = \frac{dv}{dt} = -100$$

$$ds = v\,dt$$

$$\int_{0.5}^s ds = \int_{0.1}^t (-100t + 20)\text{dt}$$

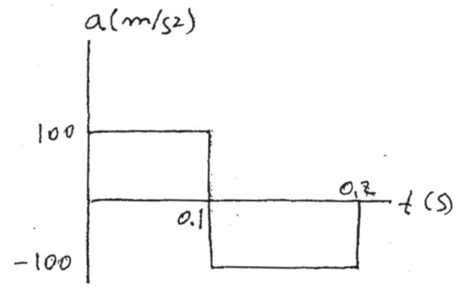
$$s - 0.5 = (-50\,t^2 + 20\,t - 1.5)$$

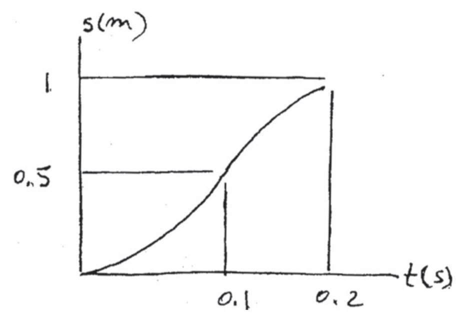
$$s = -50\,t^2 + 20\,t - 1$$

When $t = 0.2$ s,

$$s = 1 \text{ m}$$

When $t = 0.1$ s, $s = 0.5$ m and a changes from 100 m/s^2 to -100 m/s^2. When t = 0.2 s, $s = 1$ m.

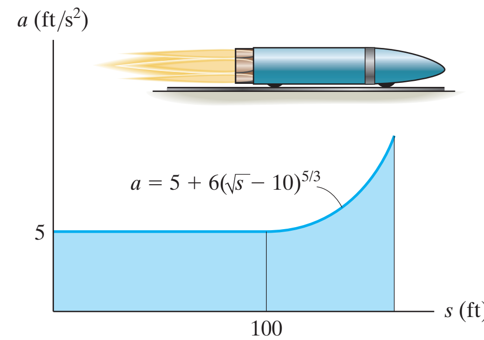




Ans:
When $t = 0.1$ s,
$s = 0.5$ m and a changes from
100 m/s^2 to -100 m/s^2. When $t = 0.2$ s,
$s = 1$ m.

12–46.

The a–s graph for a rocket moving along a straight track has been experimentally determined. If the rocket starts at $s = 0$ when $v = 0$, determine its speed when it is at $s = 75$ ft, and 125 ft, respectively. Use Simpson's rule with $n = 100$ to evaluate v at $s = 125$ ft.



SOLUTION

$0 \leq s < 100$

$$\int_0^v v\,dv = \int_0^s 5\,ds$$

$$\frac{1}{2}v^2 = 5\,\text{s}$$

$$v = \sqrt{10\,\text{s}}$$

At $s = 75$ ft, $\quad v = \sqrt{750} = 27.4$ ft/s **Ans.**

At $s = 100$ ft, $\quad v = 31.623$

$$v\,dv = a\,ds$$

$$\int_{31.623}^{v} v\,dv = \int_{100}^{125}\left[5 + 6(\sqrt{s} - 10)^{5/3}\right]ds$$

$$\frac{1}{2}v^2\Big|_{31.623}^{v} = 201.0324$$

$v = 37.4$ ft/s **Ans.**

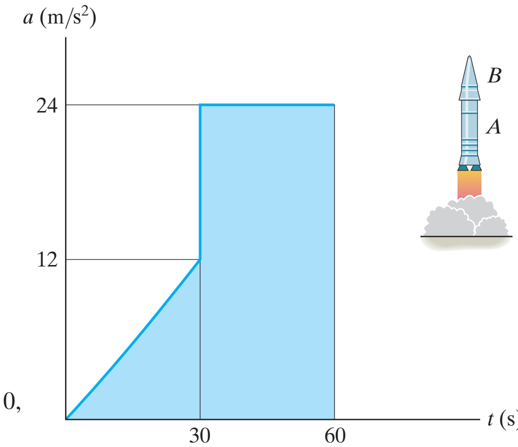
Ans:

$$v\Big|_{s=75\text{ ft}} = 27.4\text{ ft/s}$$

$$v\Big|_{s=125\text{ ft}} = 37.4\text{ ft/s}$$

12–47.

A two-stage rocket is fired vertically from rest at $s = 0$ with the acceleration as shown. After 30 s the first stage, A, burns out and the second stage, B, ignites. Plot the v–t and s–t graphs which describe the motion of the second stage for $0 \le t \le 60$ s.



SOLUTION

***v*–*t* Graph.** The $v-t$ function can be determined by integrating $dv = a\,dt$.

For $0 \le t < 30$ s, $a = \frac{12}{30}t = \left(\frac{2}{5}t\right)$ m/s^2. Using the initial condition $v = 0$ at $t = 0$,

$$\int_0^v dv = \int_0^t \frac{2}{5} t\,dt$$

$$v = \left\{\frac{1}{5}t^2\right\} \text{ m/s} \qquad \textbf{Ans.}$$

At $t = 30$ s,

$$v\Big|_{t=30\text{ s}} = \frac{1}{5}(30^2) = 180 \text{ m/s}$$

For $30 < t \le 60$ s, $a = 24$ m/s^2. Using the initial condition $v = 180$ m/s at $t = 30$ s,

$$\int_{180\text{ m/s}}^v dv = \int_{30\text{ s}}^t 24\,dt$$

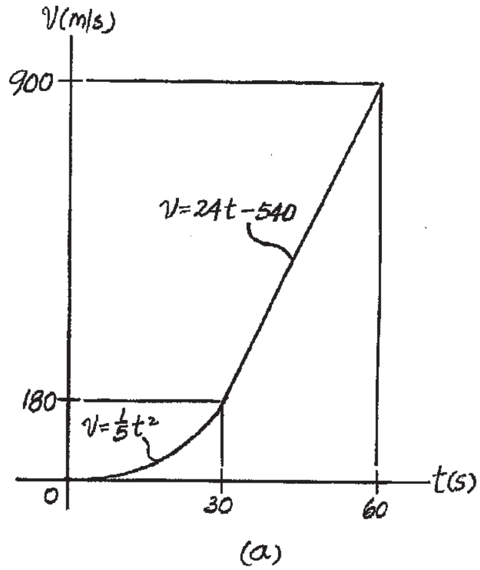
$$v - 180 = 24\,t\Big|_{30\text{ s}}^t$$

$$v = \{24t - 540\} \text{ m/s} \qquad \textbf{Ans.}$$

At $t = 60$ s,

$$v\Big|_{t=60\text{ s}} = 24(60) - 540 = 900 \text{ m/s}$$

Using these results, $v-t$ graph shown in Fig. a can be plotted.



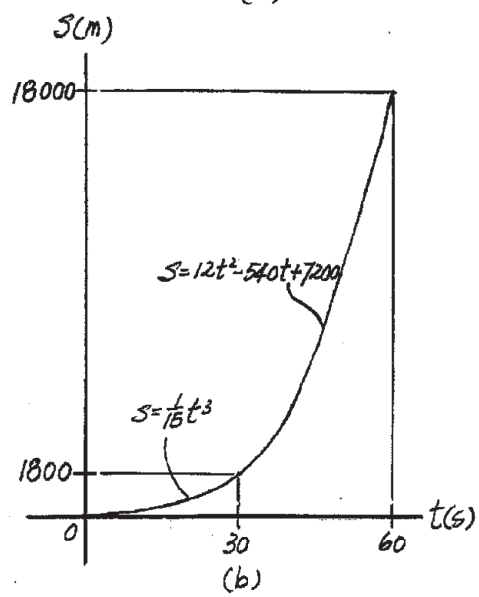
***s*–*t* Graph.** The $s-t$ function can be determined by integrating $ds = v\,dt$. For $0 \le t < 30$ s, the initial condition is $s = 0$ at $t = 0$.

$$\int_0^s ds = \int_0^t \frac{1}{5}t^2 dt$$

$$s = \left\{\frac{1}{15}t^3\right\} \text{m} \qquad \textbf{Ans.}$$

At $t = 30$ s,

$$s\Big|_{t=30\text{ s}} = \frac{1}{15}(30^3) = 1800 \text{ m}$$



12–47. Continued

For 30 s $< t \leq$ 60 s, the initial condition is $s = 1800$ m at $t = 30$ s.

$$\int_{1800\text{ m}}^{s} ds = \int_{30\text{ s}}^{t} (24t - 540)dt$$

$$s - 1800 = (12t^2 - 540t)\Big|_{30\text{ s}}^{t}$$

$$s = \{12t^2 - 540t + 7200\}\text{ m}$$

At $t = 60$ s,

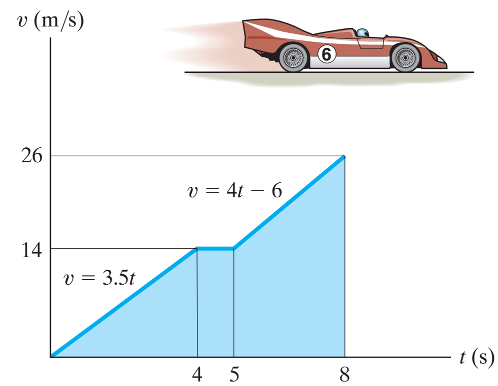
$$s\Big|_{t = 60\text{ s}} = 12(60^2) - 540(60) + 7200 = 18000\text{ m}$$

Using these results, the $s-t$ graph in Fig. b can be plotted.

Ans:
For $0 \leq t < 30$ s,
$v = \left\{\frac{1}{5}t^2\right\}$ m/s
$s = \left\{\frac{1}{15}t^3\right\}$ m
For $30 \leq t \leq 60$ s,
$v = \{24t - 540\}$ m/s
$s = \{12t^2 - 540t + 7200\}$ m

***12–48.**

The race car starts from rest and travels along a straight road until it reaches a speed of 26 m/s in 8 s as shown on the v–t graph. The flat part of the graph is caused by shifting gears. Draw the a–t graph and determine the maximum acceleration of the car.



SOLUTION

For $0 \leq t < 4$ s

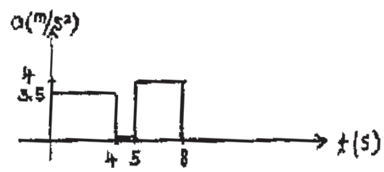
$$a = \frac{\Delta v}{\Delta t} = \frac{14}{4} = 3.5 \text{ m/s}^2$$

For $4 \text{ s} \leq t < 5$ s

$$a = \frac{\Delta v}{\Delta t} = 0$$

For $5 \text{ s} \leq t < 8$ s

$$a = \frac{\Delta v}{\Delta t} = \frac{26 - 14}{8 - 5} = 4 \text{ m/s}^2$$

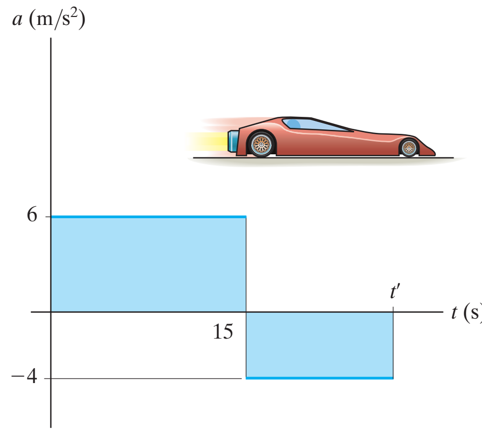


$a_{max} = 4.00 \text{ m/s}^2$ **Ans.**

Ans:
$a_{max} = 4.00 \text{ m/s}^2$

12–49.

The jet car is originally traveling at a velocity of 10 m/s when it is subjected to the acceleration shown. Determine the car's maximum velocity and the time t' when it stops. When $t = 0, s = 0$.



SOLUTION

***v–t* Function.** The $v-t$ function can be determined by integrating $dv = a\,dt$. For $0 \leq t < 15$ s, $a = 6$ m/s². Using the initial condition $v = 10$ m/s at $t = 0$,

$$\int_{10\text{ m/s}}^{v} dv = \int_0^t 6dt$$

$$v - 10 = 6t$$

$$v = \{6t + 10\}\text{ m/s}$$

The maximum velocity occurs when $t = 15$ s. Then

$$v_{\max} = 6(15) + 10 = 100\text{ m/s}$$ **Ans.**

For 15 s $< t \leq t'$, $a = -4$ m/s, Using the initial condition $v = 100$ m/s at $t = 15$ s,

$$\int_{100\text{ m/s}}^{v} dv = \int_{15\text{ s}}^{t} -4dt$$

$$v - 100 = (-4t)\Big|_{15\text{ s}}^{t}$$

$$v = \{-4t + 160\}\text{ m/s}$$

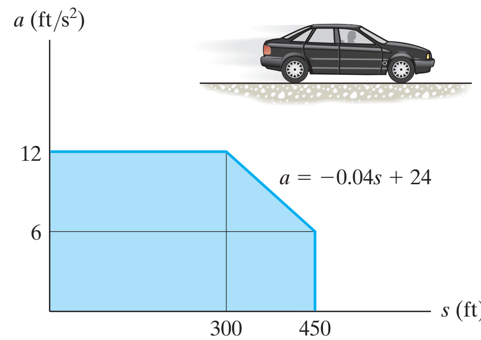
It is required that $v = 0$ at $t = t'$. Then

$$0 = -4t' + 160 \qquad t' = 40\text{ s}$$ **Ans.**

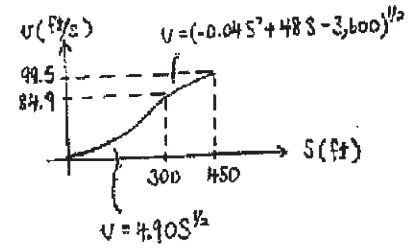
Ans:
$v_{\max} = 100$ m/s
$t' = 40$ s

12–50.

The car starts from rest at $s = 0$ and is subjected to an acceleration shown by the a–s graph. Draw the v–s graph and determine the time needed to travel 200 ft.



SOLUTION



For $s < 300$ ft

$a\,ds = v\,dv$

$$\int_0^s 12\,ds = \int_0^v v\,dv$$

$$12\text{ s} = \frac{1}{2}v^2$$

$v = 4.90\text{ s}^{1/2}$

At $s = 300$ ft, $\quad v = 84.85$ ft/s

For 300 ft $< s < 450$ ft

$a\,ds = v\,dv$

$$\int_{300}^{s}(24 - 0.04\,s)\,ds = \int_{84.85}^{v} v\,dv$$

$$24\,s - 0.02\,s^2 - 5400 = 0.5\,v^2 - 3600$$

$$v = (-0.04\,s^2 + 48\,s - 3600)^{1/2}$$

At $s = 450$ ft, $\quad v = 99.5$ ft/s

$v = 4.90\text{ s}^{1/2}$

$$\frac{ds}{dt} = 4.90\text{ s}^{1/2}$$

$$\int_0^{200} s^{-1/2}\,ds = \int_0^t 4.90\,dt$$

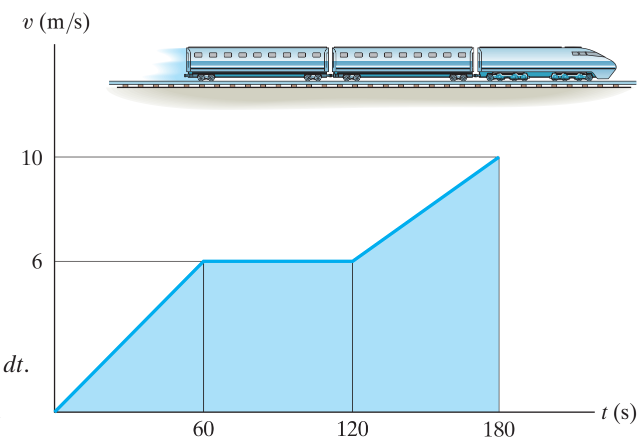
$$2\,s^{1/2}\Big|_0^{200} = 4.90\,t$$

$t = 5.77$ s **Ans.**

Ans:
For $0 \le s < 300$ ft,
$v = \{4.90\,s^{1/2}\}$ m/s.
For 300 ft $< s \le 450$ ft,
$v = \{(-0.04s^2 + 48s - 3600)^{1/2}\}$ m/s.
$s = 200$ ft when $t = 5.77$ s.

12–51.

The v–t graph for a train has been experimentally determined. From the data, construct the s–t and a–t graphs for the motion for $0 \leq t \leq 180$ s. When $t = 0$, $s = 0$.



SOLUTION

***s*–*t* Graph.** The s–t function can be determined by integrating $ds = v\,dt$. For $0 \leq t < 60$ s, $v = \frac{6}{60}t = \left(\frac{1}{10}t\right)$ m/s. Using the initial condition $s = 0$ at $t = 0$,

$$\int_0^s ds = \int_0^t \left(\frac{1}{10}t\right)dt$$

$$s = \left\{\frac{1}{20}t^2\right\} \text{ m} \qquad \textbf{Ans.}$$

When $t = 60$ s,

$$s|_{t-60\text{ s}} = \frac{1}{20}(60^2) = 180 \text{ m}$$

For $60 \text{ s} < t < 120$ s, $v = 6$ m/s. Using the initial condition $s = 180$ m at $t = 60$ s,

$$\int_{180\text{ m}}^s ds = \int_{60\text{ s}}^t 6\,dt$$

$$s - 180 = 6t\Big|_{60\text{ s}}^{t}$$

$$s = \{6t - 180\} \text{ m} \qquad \textbf{Ans.}$$

At $t = 120$ s,

$$s|_{t-120\text{ s}} = 6(120) - 180 = 540 \text{ m}$$

For $120 \text{ s} < t \leq 180$ s, $\frac{v - 6}{t - 120} = \frac{10 - 6}{180 - 120}$; $v = \left\{\frac{1}{15}t - 2\right\}$ m/s. Using the initial condition $s = 540$ m at $t = 120$ s,

$$\int_{540\text{ m}}^s ds = \int_{120\text{ s}}^t \left(\frac{1}{15}\text{t} - 2\right)dt$$

$$s - 540 = \left(\frac{1}{30}t^2 - 2t\right)\Big|_{120\text{ s}}^{t}$$

$$s = \left\{\frac{1}{30}t^2 - 2t + 300\right\} \text{ m} \qquad \textbf{Ans.}$$

At $t = 180$ s,

$$s|_{t=180\text{ s}} = \frac{1}{30}(180^2) - 2(180) + 300 = 1020 \text{ m}$$

Using these results, s–t graph shown in Fig. *a* can be plotted.

12–51. Continued

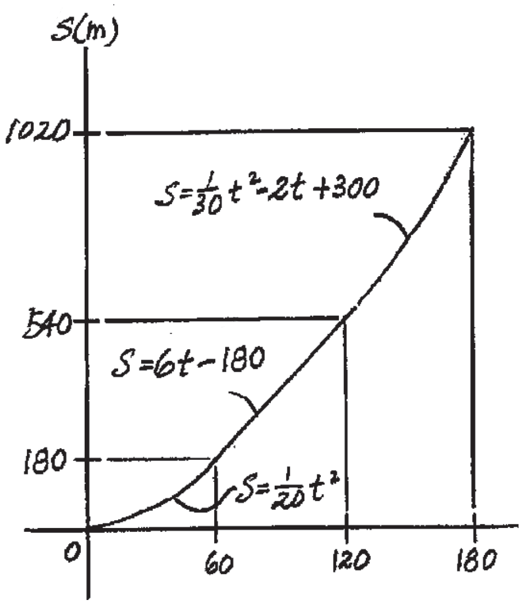
***a–t* Graph.** The a–t function can be determined using $a = \dfrac{dv}{dt}$.

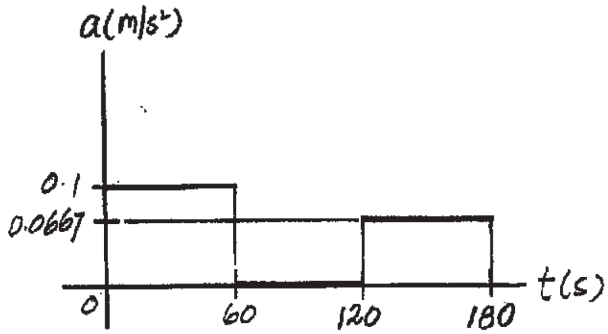
For $0 \le t < 60$ s, $\quad a = \dfrac{d\left(\frac{1}{10}t\right)}{dt} = 0.1\ \text{m/s}^2$ **Ans.**

For $60\ \text{s} < t < 120$ s, $\quad a = \dfrac{d(6)}{dt} = 0$ **Ans.**

For $120\ \text{s} < t \le 180$ s, $\quad a = \dfrac{d\left(\frac{1}{15}t - 2\right)}{dt} = 0.0667\ \text{m/s}^2$ **Ans.**

Using these results, a–t graph shown in Fig. b can be plotted.

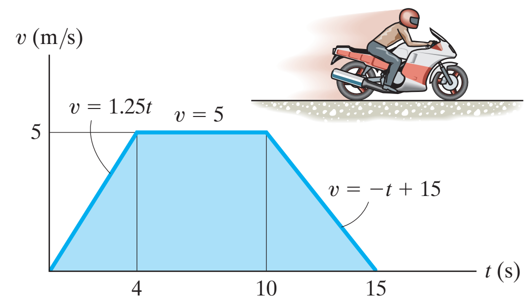




Ans:
For $0 \le t < 60$ s,
$s = \left\{\dfrac{1}{20}t^2\right\}$ m,
$a = 0.1\ \text{m/s}^2$.
For $60\ \text{s} < t < 120$ s,
$s = \{6t - 180\}$ m,
$a = 0$. For $120\ \text{s} < t \le 180$ s,
$s = \left\{\dfrac{1}{30}t^2 - 2t + 300\right\}$ m,
$a = 0.0667\ \text{m/s}^2$.

***12–52.**

A motorcycle starts from rest at $s = 0$ and travels along a straight road with the speed shown by the v–t graph. Determine the total distance the motorcycle travels until it stops when $t = 15$ s. Also plot the a–t and s–t graphs.



SOLUTION

For $t < 4$ s

$$a = \frac{dv}{dt} = 1.25$$

$$\int_0^s ds = \int_0^t 1.25\, t\, dt$$

$$s = 0.625\, t^2$$

When $t = 4$ s, $s = 10$ m

For 4 s $< t <$ 10 s

$$a = \frac{dv}{dt} = 0$$

$$\int_{10}^s ds = \int_4^t 5\, dt$$

$$s = 5\,t - 10$$

When $t = 10$ s, $s = 40$ m

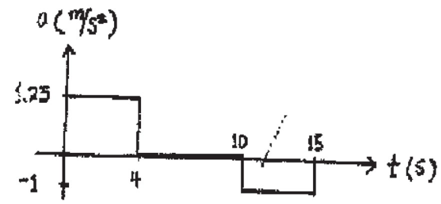
For 10 s $< t <$ 15 s

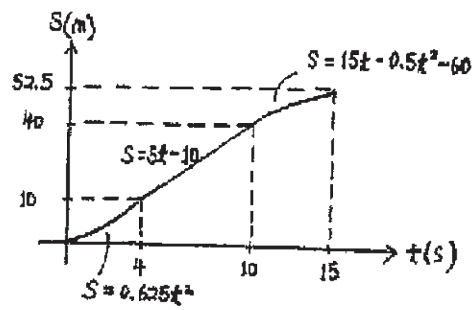
$$a = \frac{dv}{dt} = -1$$

$$\int_{40}^s ds = \int_{10}^t (15 - t)\, dt$$

$$s = 15\,t - 0.5\,t^2 - 60$$

When $t = 15$ s, $s = 52.5$ m **Ans.**

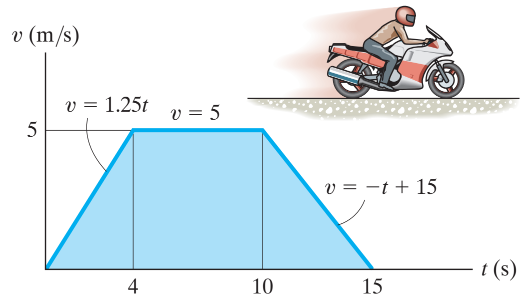




Ans:
When $t = 15$ s, $s = 52.5$ m

12–53.

A motorcycle starts from rest at $s = 0$ and travels along a straight road with the speed shown by the v–t graph. Determine the motorcycle's acceleration and position when $t = 8$ s and $t = 12$ s.



SOLUTION

At $t = 8$ s

$$a = \frac{dv}{dt} = 0$$ **Ans.**

$$\Delta s = \int v\, dt$$

$$s - 0 = \frac{1}{2}(4)(5) + (8 - 4)(5) = 30$$

$s = 30$ m **Ans.**

At $t = 12$ s

$$a = \frac{dv}{dt} = \frac{-5}{5} = -1 \text{ m/s}^2$$ **Ans.**

$$\Delta s = \int v\, dt$$

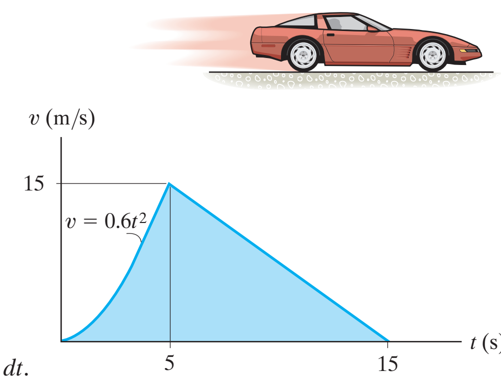
$$s - 0 = \frac{1}{2}(4)(5) + (10 - 4)(5) + \frac{1}{2}(15 - 10)(5) - \frac{1}{2}\left(\frac{3}{5}\right)(5)\left(\frac{3}{5}\right)(5)$$

$s = 48$ m **Ans.**

Ans:
At $t = 8$ s,
$a = 0$ and $s = 30$ m.
At $t = 12$ s,
$a = -1 \text{ m/s}^2$
and $s = 48$ m.

12–54.

The v–t graph for the motion of a car as it moves along a straight road is shown. Draw the s–t and a–t graphs. Also determine the average speed and the distance traveled for the 15-s time interval. When $t = 0$, $s = 0$.



SOLUTION

***s*–*t* Graph.** The s–t function can be determined by integrating $ds = v\,dt$. For $0 \le t < 5$ s, $v = 0.6t^2$. Using the initial condition $s = 0$ at $t = 0$,

$$\int_0^s ds = \int_0^t 0.6t^2 dt$$

$$s = \{0.2t^3\}\text{ m} \qquad \textbf{Ans.}$$

At $t = 5$ s,

$$s|_{t=5\text{ s}} = 0.2(5^3) = 25\text{ m}$$

For $5\text{ s} < t \le 15$ s, $\dfrac{v - 15}{t - 5} = \dfrac{0 - 15}{15 - 5}$; $v = \dfrac{1}{2}(45 - 3t)$. Using the initial condition $s = 25$ m at $t = 5$ s,

$$\int_{25\text{ m}}^s ds = \int_{5\text{ s}}^t \frac{1}{2}(45 - 3t)dt$$

$$s - 25 = \frac{45}{2}t - \frac{3}{4}t^2 - 93.75$$

$$s = \left\{\frac{1}{4}(90t - 3t^2 - 275)\right\}\text{ m} \qquad \textbf{Ans.}$$

At $t = 15$ s,

$$s = \frac{1}{4}\left[90(15) - 3(15^2) - 275\right] = 100\text{ m} \qquad \textbf{Ans.}$$

Thus the average speed is

$$v_{\text{avg}} = \frac{s_T}{t} = \frac{100\text{ m}}{15\text{ s}} = 6.67\text{ m/s} \qquad \textbf{Ans.}$$

using these results, the s–t graph shown in Fig. a can be plotted.

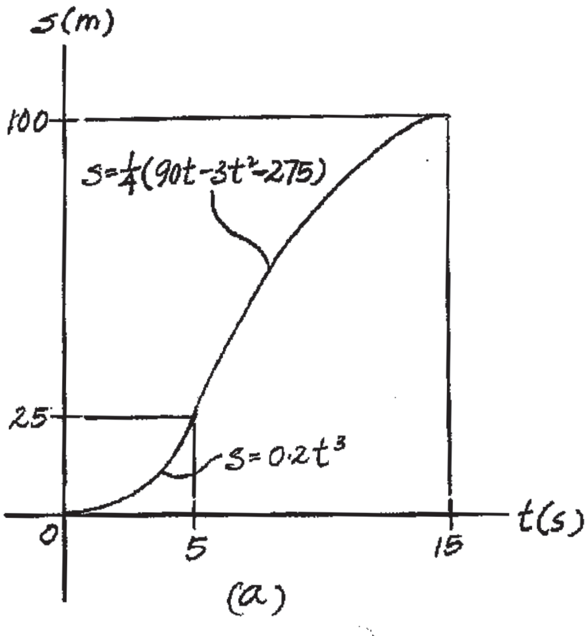
12–54. Continued

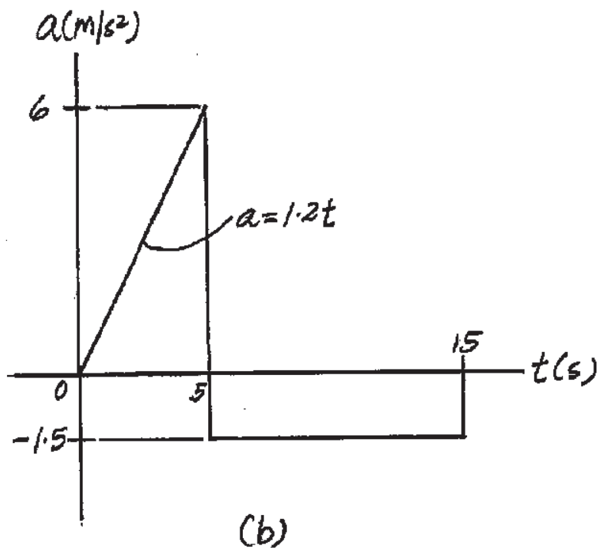
***a–t* Graph.** The *a–t* function can be determined using $a = \frac{dv}{dt}$.

For $0 \le t < 5$ s, $a = \frac{d(0.6\,t^2)}{dt} = \{1.2\,t\}$ m/s^2 **Ans.**

At $t = 5$ s, $a = 1.2(5) = 6$ m/s^2 **Ans.**

For 5 s $< t \le 15$ s, $a = \frac{d\left[\frac{1}{2}(45 - 3t)\right]}{dt} = -1.5$ m/s^2 **Ans.**

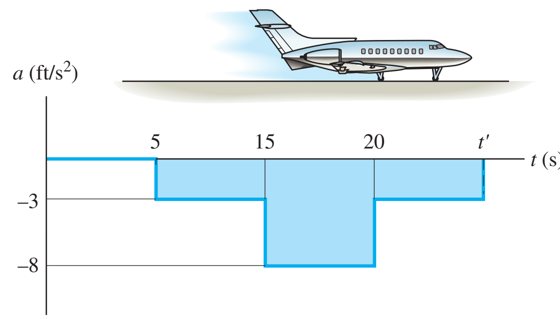




Ans:
For $0 \le t < 5$ s,
$s = \{0.2t^3\}$ m
$a = \{1.2t\}$ m/s^2
For 5 s $< t \le 15$ s,
$s = \left\{\frac{1}{4}(90t - 3t^2 - 275)\right\}$ m
$a = -1.5$ m/s^2
At $t = 15$ s,
$s = 100$ m
$v_{avg} = 6.67$ m/s

12–55.

An airplane lands on the straight runway, originally traveling at 110 ft/s when $s = 0$. If it is subjected to the decelerations shown, determine the time t' needed to stop the plane and construct the s–t graph for the motion.



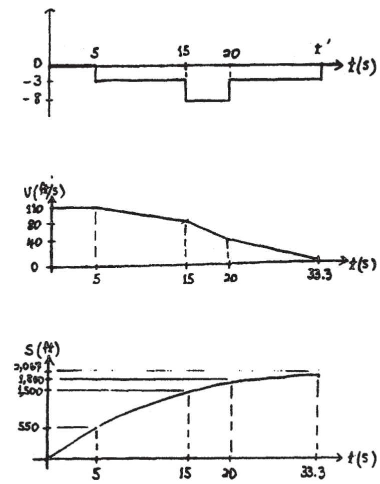
SOLUTION

$v_0 = 110$ ft/s

$\Delta v = \int a\, dt$

$0 - 110 = -3(15 - 5) - 8(20 - 15) - 3(t' - 20)$

$t' = 33.3$ s **Ans.**



$s\big|_{t = 5\text{s}} = 550$ ft

$s\big|_{t = 15\text{s}} = 1500$ ft

$s\big|_{t = 20\text{s}} = 1800$ ft

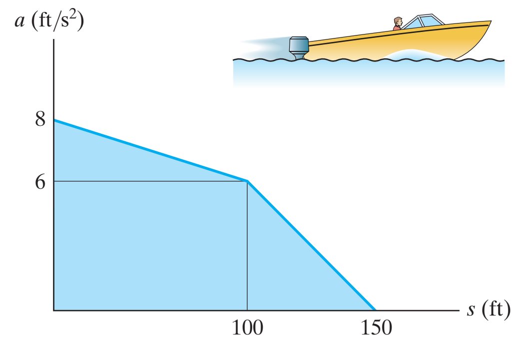
$s\big|_{t = 33.3\text{s}} = 2067$ ft

Ans:

$t' = 33.3$ s
$s\big|_{t=5\text{ s}} = 550$ ft
$s\big|_{t=15\text{ s}} = 1500$ ft
$s\big|_{t=20\text{ s}} = 1800$ ft
$s\big|_{t=33.3\text{ s}} = 2067$ ft

***12–56.**

Starting from rest at $s = 0$, a boat travels in a straight line with the acceleration shown by the a–s graph. Determine the boat's speed when $s = 50$ ft, 100 ft, and 150 ft.



SOLUTION

v–s **Function.** The $v-s$ function can be determined by integrating $v\,dv = a\,ds$. For $0 \le s < 100$ ft, $\dfrac{a-8}{s-0} = \dfrac{6-8}{100-0}$, $a = \left\{-\dfrac{1}{50}s + 8\right\}$ ft/s^2. Using the initial condition $v-0$ at $s = 0$,

$$\int_0^v v\,dv = \int_0^s \left(-\frac{1}{50}s + 8\right) ds$$

$$\frac{v^2}{2}\Bigg|_0^v = \left(-\frac{1}{100}s^2 + 8\,s\right)\Bigg|_0^s$$

$$\frac{v^2}{2} = 8s - \frac{1}{100}s^2$$

$$v = \left\{\sqrt{\frac{1}{50}}\,(800\,s - s^2)\right\} \text{ ft/s}$$

At $s = 50$ ft,

$$v|_{s=50\text{ ft}} = \sqrt{\frac{1}{50}[800\,(50) - 50^2]} = 27.39 \text{ ft/s} = 27.4 \text{ ft/s} \qquad \textbf{Ans.}$$

At $s = 100$ ft,

$$v|_{s=100\text{ ft}} = \sqrt{\frac{1}{50}[800\,(100) - 100^2]} = 37.42 \text{ ft/s} = 37.4 \text{ ft/s} \qquad \textbf{Ans.}$$

For $100 \text{ ft} < s \le 150$ ft, $\dfrac{a-0}{s-150} = \dfrac{6-0}{100-150}$; $a = \left\{-\dfrac{3}{25}s + 18\right\}$ ft/s^2. Using the initial condition $v = 37.42$ ft/s at $s = 100$ ft,

$$\int_{37.42\text{ ft/s}}^{v} v\,dv = \int_{100\text{ ft}}^{s} \left(-\frac{3}{25}s + 18\right) ds$$

$$\frac{v^2}{2}\Bigg|_{37.42\text{ ft/s}}^{v} = \left(-\frac{3}{50}s^2 + 18s\right)\Bigg|_{100\text{ ft}}^{s}$$

$$v = \left\{\frac{1}{5}\sqrt{-3s^2 + 900s - 25000}\right\} \text{ ft/s}$$

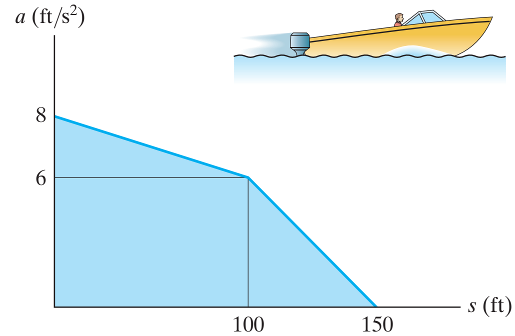
At $s = 150$ ft

$$v|_{s=150\text{ ft}} = \frac{1}{5}\sqrt{-3(150^2) + 900\,(150) - 25000} = 41.23 \text{ ft/s} = 41.2 \text{ ft/s} \qquad \textbf{Ans.}$$

Ans:
$v|_{s=50\text{ ft}} = 27.4$ ft/s
$v|_{s=100\text{ ft}} = 37.4$ ft/s
$v|_{s=150\text{ ft}} = 41.2$ ft/s

12–57.

Starting from rest at $s = 0$, a boat travels in a straight line with the acceleration shown by the a–s graph. Construct the v–s graph.



SOLUTION

v–s **Graph.** The $v-s$ function can be determined by integrating $v\,dv = a\,ds$. For $0 \le s < 100$ ft, $\frac{a-8}{s-0} = \frac{6-8}{100-0}$, $a = \left\{-\frac{1}{50}s + 8\right\}$ ft/s² using the initial condition $v = 0$ at $s = 0$,

$$\int_0^v v\,dv = \int_0^s \left(-\frac{1}{50}s + 8\right) ds$$

$$\frac{v^2}{2}\Big|_0^v = \left(-\frac{1}{100}s^2 + 8\,s\right)\Big|_0^s$$

$$\frac{v^2}{2} = 8\,s - \frac{1}{100}s^2$$

$$v = \left\{\sqrt{\frac{1}{50}(800\,s - s^2)}\right\} \text{ ft/s}$$

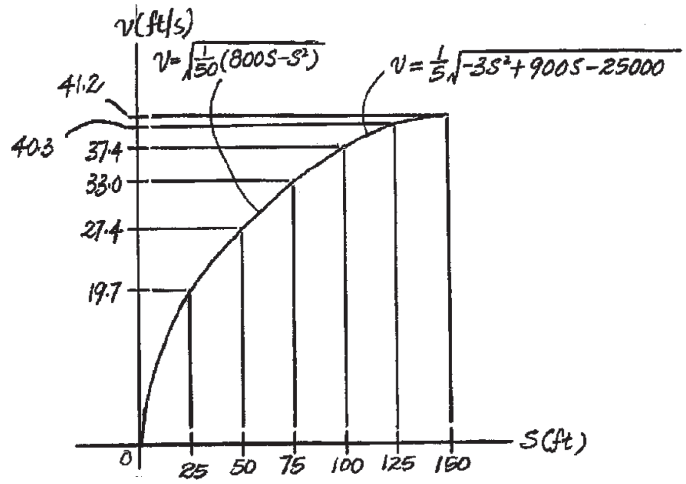
At $s = 25$ ft, 50 ft, 75 ft and 100 ft

$$v|_{s=25\text{ ft}} = \sqrt{\frac{1}{50}[800\,(25) - 25^2]} = 19.69 \text{ ft/s}$$

$$v|_{s=50\text{ ft}} = \sqrt{\frac{1}{50}[800\,(50) - 50^2]} = 27.39 \text{ ft/s}$$

$$v|_{s=75\text{ ft}} = \sqrt{\frac{1}{50}[800\,(75) - 75^2]} = 32.98 \text{ ft/s}$$

$$v|_{s=100\text{ ft}} = \sqrt{\frac{1}{50}[800\,(100) - 100^2]} = 37.42 \text{ ft/s}$$



For 100 ft $< s \le 150$ ft, $\frac{a-0}{s-150} = \frac{6-0}{100-150}$; $a = \left\{-\frac{3}{25}s + 18\right\}$ ft/s² using the initial condition $v = 37.42$ ft/s at $s = 100$ ft,

$$\int_{37.42\text{ ft/s}}^v v\,dv = \int_{100\text{ ft}}^s \left(-\frac{3}{25}s + 18\right) ds$$

$$\frac{v^2}{2}\Big|_{37.42\text{ ft/s}}^v = \left(-\frac{3}{50}s^2 + 18\,s\right)\Big|_{100\text{ ft}}^s$$

$$v = \left\{\frac{1}{5}\sqrt{-3s^2 + 900s - 25000}\right\} \text{ ft/s}$$

At $s = 125$ ft and $s = 150$ ft

$$v|_{s=125\text{ ft}} = \frac{1}{5}\sqrt{-3(125^2) + 900\,(125) - 25000} = 40.31 \text{ ft/s}$$

$$v|_{s=150\text{ ft}} = \frac{1}{5}\sqrt{-3(150^2) + 900\,(150) - 25000} = 41.23 \text{ ft/s}$$

Ans:

For $0 \le s < 100$ ft,

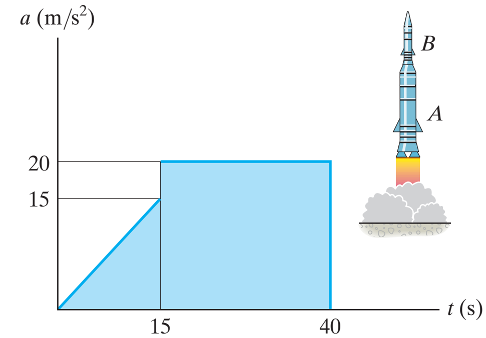
$$v = \left\{\sqrt{\frac{1}{50}(800s - s^2)}\right\} \text{ ft/s}$$

For 100 ft $< s \le 150$ ft,

$$v = \left\{\frac{1}{5}\sqrt{-3s^2 + 900s - 25000}\right\} \text{ ft/s}$$

12–58.

A two-stage rocket is fired vertically from rest with the acceleration shown. After 15 s the first stage A burns out and the second stage B ignites. Plot the v–t and s–t graphs which describe the motion of the second stage for $0 \leq t \leq 40$ s.



SOLUTION

For $0 \leq t < 15$

$a = t$

$$\int_0^v dv = \int_0^t t\,dt$$

$$v = \frac{1}{2}t^2$$

$v = 112.5$ when $t = 15$ s

$$\int_0^s ds = \int_0^t \frac{1}{2}t^2\,dt$$

$$s = \frac{1}{6}t^3$$

$s = 562.5$ when $t = 15$ s

For $15 < t < 40$

$a = 20$

$$\int_{112.5}^v dv = \int_{1.5}^t 20\,dt$$

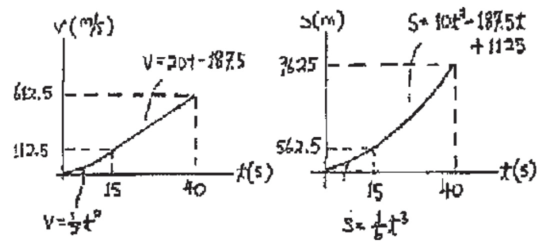
$$v = 20t - 187.5$$

$v = 612.5$ when $t = 40\,s$

$$\int_{562.5}^s ds = \int_{15}^t (20\,t - 187.5)\,dt$$

$$s = 10\,t^2 - 187.5\,t + 1125$$

$s = 9625$ when $t = 40$ s



Ans:

For $0 \leq t < 15$ s,

$$v = \left\{\frac{1}{2}t^2\right\}\text{m/s}$$

$$s = \left\{\frac{1}{6}t^3\right\}\text{m}$$

For $15\text{ s} < t \leq 40$ s,

$v = \{20t - 187.5\text{ m/s}\}$

$s = \{10t^2 - 187.5t + 1125\}$ m

12–59.

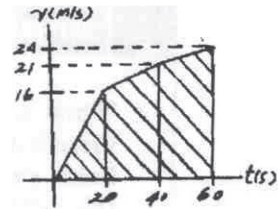
The speed of a train during the first minute has been recorded as follows:

t (s)	0	20	40	60
v (m/s)	0	16	21	24

Plot the $v-t$ graph, approximating the curve as straight-line segments between the given points. Determine the total distance traveled.

SOLUTION

The total distance traveled is equal to the area under the graph.



$$s_T = \frac{1}{2}(20)(16) + \frac{1}{2}(40 - 20)(16 + 21) + \frac{1}{2}(60 - 40)(21 + 24) = 980 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$s_T = 980$ m

***12–60.**

A man riding upward in a freight elevator accidentally drops a package off the elevator when it is 100 ft from the ground. If the elevator maintains a constant upward speed of 4 ft/s, determine how high the elevator is from the ground the instant the package hits the ground. Draw the v–t curve for the package during the time it is in motion. Assume that the package was released with the same upward speed as the elevator.

SOLUTION

For package:

$(+\uparrow)\qquad v^2 = v_0^2 + 2a_c(s_2 - s_0)$

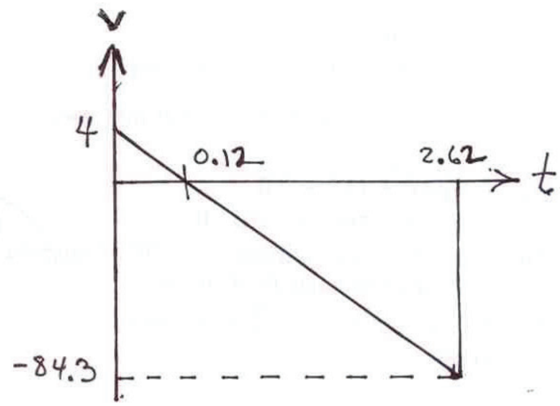
$$v^2 = (4)^2 + 2(-32.2)(0 - 100)$$

$$v = 80.35 \text{ ft/s} \downarrow$$

$(+\uparrow)\qquad v = v_0 + a_c t$

$$-80.35 = 4 + (-32.2)t$$

$$t = 2.620 \text{ s}$$



For elevator:

$(+\uparrow)\qquad s_2 = s_0 + vt$

$$s = 100 + 4(2.620)$$

$$s = 110 \text{ ft}$$

Ans.

Ans:
$s = 110$ ft

12–61.

Two cars start from rest side by side and travel along a straight road. Car A accelerates at $4\ \text{m/s}^2$ for 10 s and then maintains a constant speed. Car B accelerates at $5\ \text{m/s}^2$ until reaching a constant speed of 25 m/s and then maintains this speed. Construct the a–t, v–t, and s–t graphs for each car until $t = 15$ s. What is the distance between the two cars when $t = 15$ s?

SOLUTION

Car A:

$$v = v_0 + a_c t$$

$$v_A = 0 + 4t$$

At $t = 10$ s, $\quad v_A = 40$ m/s

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_A = 0 + 0 + \frac{1}{2}(4)t^2 = 2t^2$$

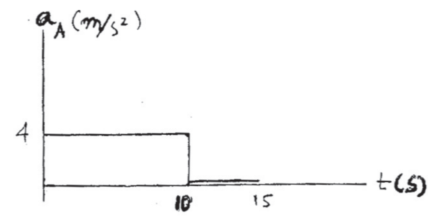
At $t = 10$ s, $\qquad s_A = 200$ m

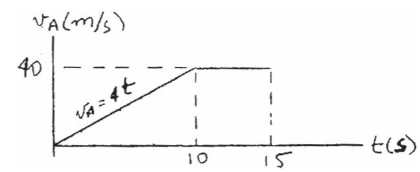
$t > 10$ s, $\qquad ds = v\, dt$

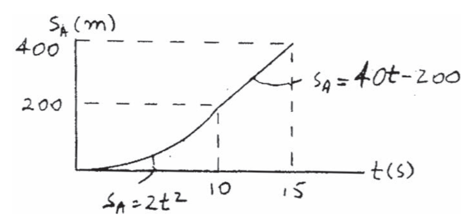
$$\int_{200}^{s_A} ds = \int_{10}^{t} 40\, dt$$

$$s_A = 40t - 200$$

At $t = 15$ s, $\qquad s_A = 400$ m







Car B:

$$v = v_0 + a_c t$$

$$v_B = 0 + 5t$$

When $v_B = 25$ m/s, $\qquad t = \frac{25}{5} = 5$ s

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s_B = 0 + 0 + \frac{1}{2}(5)t^2 = 2.5t^2$$

When $t = 10$ s, $v_A = (v_A)_{max} = 40$ m/s and $s_A = 200$ m.

When $t = 5$ s, $s_B = 62.5$ m.

When $t = 15$ s, $s_A = 400$ m and $s_B = 312.5$ m.

12–61. Continued

At $t = 5$ s, $\quad s_B = 62.5$ m

$t > 5$ s, $\quad ds = v\,dt$

$$\int_{62.5}^{s_B} ds = \int_5^t 25\,dt$$

$$s_B - 62.5 = 25t - 125$$

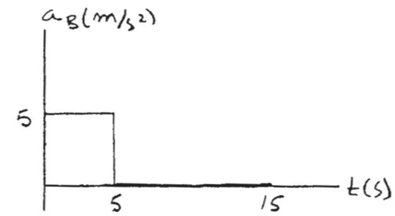
$$s_B = 25t - 62.5$$

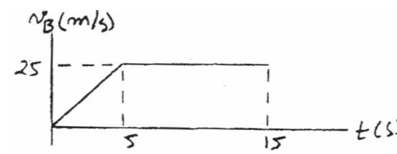
When $t = 15$ s, $\quad s_B = 312.5$

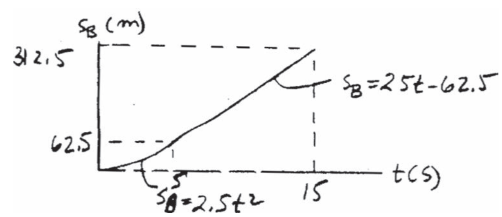
Distance between the cars is

$\Delta s = s_A - s_B = 400 - 312.5 = 87.5$ m **Ans.**

Car A is ahead of car B.





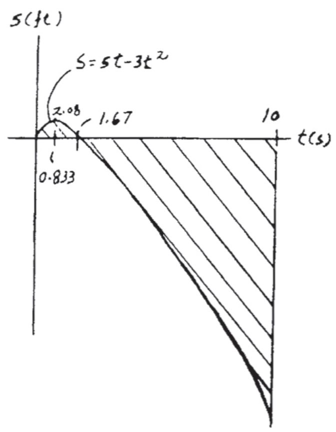


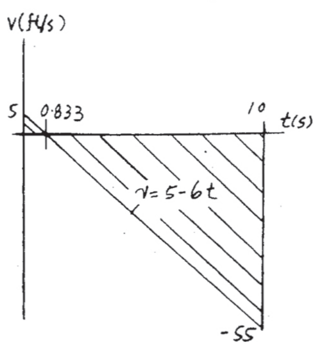
Ans:
When $t = 5$ s,
$s_B = 62.5$ m.
When $t = 10$ s,
$v_A = (v_A)_{max} = 40$ m/s and
$s_A = 200$ m.
When $t = 15$ s,
$s_A = 400$ m and $s_B = 312.5$ m.
$\Delta s = s_A - s_B = 87.5$ m

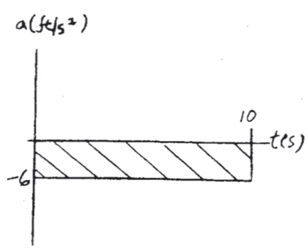
12–62.

If the position of a particle is defined as $s = (5t - 3t^2)$ ft, where t is in seconds, construct the s–t, v–t, and a–t graphs for $0 \leq t \leq 10$ s.

SOLUTION



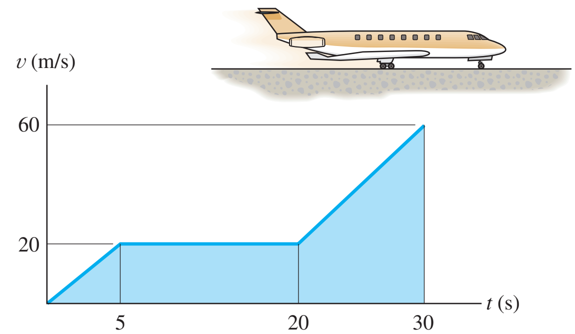




Ans:
$v = \{5 - 6t\}$ ft/s
$a = -6$ ft/s^2

12–63.

From experimental data, the motion of a jet plane while traveling along a runway is defined by the $\nu - t$ graph. Construct the $s - t$ and $a - t$ graphs for the motion. When $t = 0, s = 0$.



SOLUTION

***s* − *t* Graph:** The position in terms of time t can be obtained by applying $v = \frac{ds}{dt}$. For time interval $\mathbf{0 \ s \leq t < 5 \ s}$, $v = \frac{20}{5}t = (4t)$ m/s.

$$ds = vdt$$

$$\int_0^s ds = \int_0^t 4t dt$$

$$s = \left(2t^2\right) \text{ m}$$

When $t = 5$ s, $\quad s = 2\left(5^2\right) = 50$ m,

For time interval $\mathbf{5 \ s < t < 20 \ s}$,

$$ds = vdt$$

$$\int_{50 \text{ m}}^s ds = \int_{5 \text{ a}}^t 20 dt$$

$$s = (20t - 50) \text{ m}$$

When $t = 20$ s, $\quad s = 20(20) - 50 = 350$ m

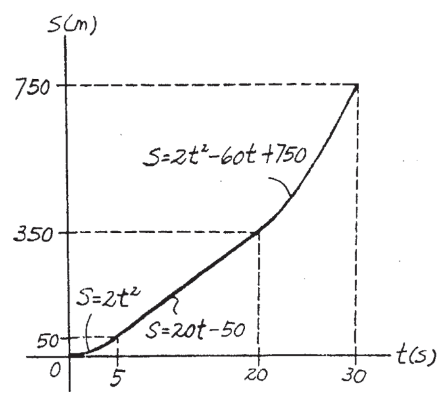
For time interval $\mathbf{20 \ s < t \leq 30 \ s}$, $\frac{v - 20}{t - 20} = \frac{60 - 20}{30 - 20}$, $v = (4t - 60)$ m/s.

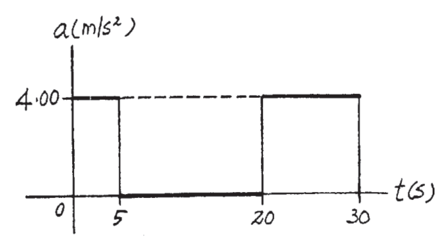
$$ds = vdt$$

$$\int_{350 \text{ m}}^s ds = \int_{20 \text{ a}}^t (4t - 60)\, dt$$

$$s = \left(2t^2 - 60t + 750\right) \text{ m}$$

When $t = 30$ s, $\quad s = 2\left(30^2\right) - 60(30) + 750 = 750$ m



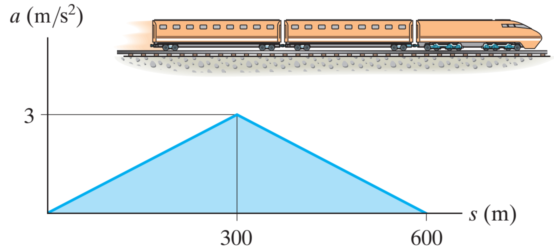


***a* − *t* Graph:** The acceleration function in terms of time t can be obtained by applying $a = \frac{dv}{dt}$. For time interval $\mathbf{0 \ s \leq t < 5 \ s}$, $\mathbf{5 \ s < t < 20 \ s}$ and $\mathbf{20 \ s < t \leq 30 \ s}$, $a = \frac{dv}{dt} = 4.00 \text{ m/s}^2$, $a = \frac{dv}{dt} = 0$ and $a = \frac{dv}{dt} = 4.00 \text{ m/s}^2$, respectively.

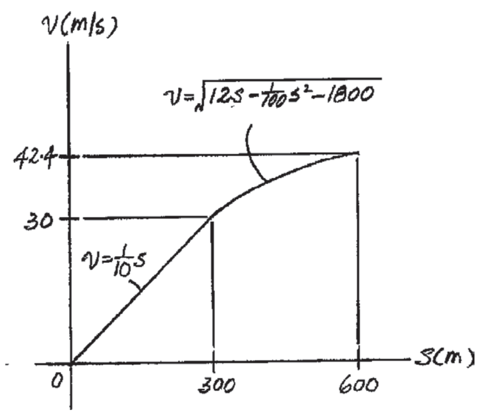
Ans:
For $0 \leq t < 5$ s,
$s = \left\{2t^2\right\}$ m and $a = 4 \text{ m/s}^2$.
For $5 \text{ s} < t < 20$ s,
$s = \{20t - 50\}$ m and $a = 0$.
For $20 \text{ s} < t \leq 30$ s,
$s = \left\{2t^2 - 60t + 750\right\}$ m
and $a = 4 \text{ m/s}^2$.

***12–64.**

The motion of a train is described by the *a–s* graph shown. Draw the *v–s* graph if $v = 0$ at $s = 0$.



SOLUTION



***v–s* Graph.** The $v-s$ function can be determined by integrating $v\,dv = a\,ds$. For $0 \le s < 300$ m, $a = \left(\frac{3}{300}\right)s = \left(\frac{1}{100}s\right)$ m/s^2. Using the initial condition $v = 0$ at $s = 0$,

$$\int_0^v v\,dv = \int_0^s \left(\frac{1}{100}s\right)ds$$

$$\frac{v^2}{2} = \frac{1}{200}s^2$$

$$v = \left\{\frac{1}{10}s\right\}\text{ m/s} \qquad \textbf{Ans.}$$

At $s = 300$ m,

$$v\big|_{s=300\text{ m}} = \frac{1}{10}(300) = 30\text{ m/s}$$

For $300\text{ m} < s \le 600$ m, $\frac{a-3}{s-300} = \frac{0-3}{600-300}$; $a = \left\{-\frac{1}{100}s + 6\right\}$ m/s^2, using the initial condition $v = 30$ m/s at $s = 300$ m,

$$\int_{30\text{ m/s}}^v v\,dv = \int_{300\text{ m}}^s \left(-\frac{1}{100}s + 6\right)ds$$

$$\frac{v^2}{2}\Big|_{30\text{ m/s}}^v = \left(-\frac{1}{200}s^2 + 6s\right)\Big|_{300\text{ m}}^s$$

$$\frac{v^2}{2} - 450 = 6s - \frac{1}{200}s^2 - 1350$$

$$v = \left\{\sqrt{12s - \frac{1}{100}s^2 - 1800}\right\}\text{ m/s} \qquad \textbf{Ans.}$$

At $s = 600$ m,

$$v = \sqrt{12(600) - \frac{1}{100}(600^2) - 1800} = 42.43\text{ m/s}$$

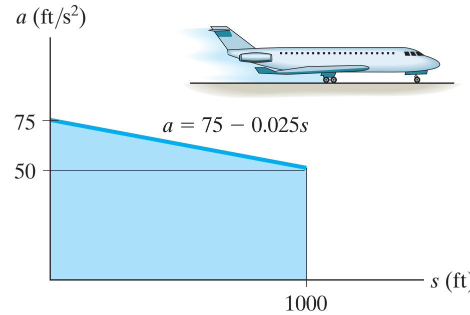
Using these results, the $v-s$ graph shown in Fig. *a* can be plotted.

Ans:

$v = \left\{\frac{1}{10}s\right\}$ m/s

12–65.

The jet plane starts from rest at $s = 0$ and is subjected to the acceleration shown. Determine the speed of the plane when it has traveled 1000 ft. Also, how much time is required for it to travel 1000 ft?



SOLUTION

v–s **Function.** Here, $\dfrac{a - 75}{s - 0} = \dfrac{50 - 75}{1000 - 0}$; $a = \{75 - 0.025s\}\ \text{ft/s}^2$. The function $v_{(s)}$ can be determined by integrating $v\,dv = a\,ds$. Using the initial condition $v = 0$ at $s = 0$,

$$\int_0^v v\,dv = \int_0^s (75 - 0.025\,s)\,ds$$

$$\frac{v^2}{2} = 75\,s - 0.0125\,s^2$$

$$v = \left\{\sqrt{150\,s - 0.025\,s^2}\right\}\ \text{ft/s}$$

At $s = 1000$ ft,

$$v = \sqrt{150\,(1000) - 0.025(1000^2)}$$

$$= 353.55\ \text{ft/s} = 354\ \text{ft/s}$$ **Ans.**

Time. t as a function of s can be determined by integrating $dt = \dfrac{ds}{v}$. Using the initial condition $s = 0$ at $t = 0$;

$$\int_0^t dt = \int_0^s \frac{ds}{\sqrt{150\,s - 0.025\,s^2}}$$

$$t = \left[-\frac{1}{\sqrt{0.025}}\sin^{-1}\left(\frac{150 - 0.05\,s}{150}\right)\right]\Bigg|_0^s$$

$$t = \frac{1}{\sqrt{0.025}}\left[\frac{\pi}{2} - \sin^{-1}\left(\frac{150 - 0.05\,s}{150}\right)\right]$$

At $s = 1000$ ft,

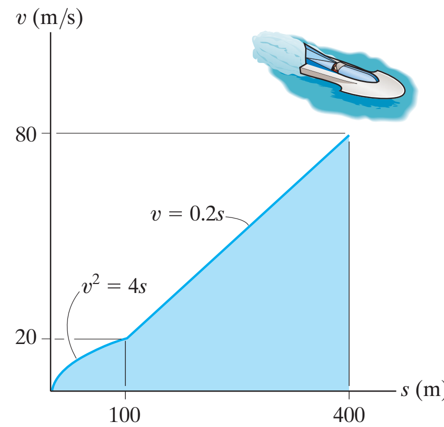
$$t = \frac{1}{\sqrt{0.025}}\left\{\frac{\pi}{2} - \sin^{-1}\left[\frac{150 - 0.05(1000)}{150}\right]\right\}$$

$$= 5.319\ \text{s} = 5.32\ \text{s}$$ **Ans.**

Ans:
$v = 354$ ft/s
$t = 5.32$ s

12–66.

The boat travels along a straight line with the speed described by the graph. Construct the s–t and $a-s$ graphs. Also, determine the time required for the boat to travel a distance $s = 400$ m if $s = 0$ when $t = 0$.



SOLUTION

$s-t$ Graph: For $0 \leq s < 100$ m, the initial condition is $s = 0$ when $t = 0$ s.

$$(\overset{+}{\rightarrow}) \qquad dt = \frac{ds}{v}$$

$$\int_0^t dt = \int_0^s \frac{ds}{2s^{1/2}}$$

$$t = s^{1/2}$$

$$s = \left(t^2\right) \text{ m}$$

When $s = 100$ m,

$$100 = t^2 \qquad t = 10 \text{ s}$$

For $100 \text{ m} < s \leq 400$ m, the initial condition is $s = 100$ m when $t = 10$ s.

$$(\overset{+}{\rightarrow}) \qquad dt = \frac{ds}{v}$$

$$\int_{10\text{ s}}^t dt = \int_{100\text{ m}}^s \frac{ds}{0.2s}$$

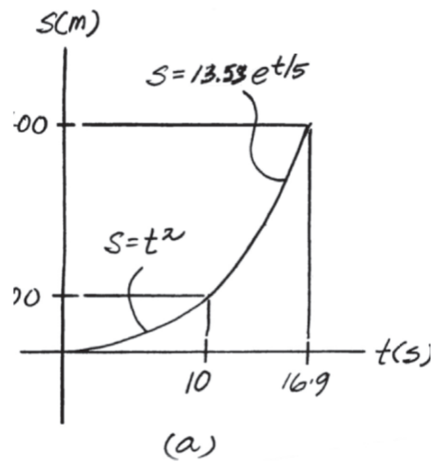
$$t - 10 = 5\ln\frac{s}{100}$$

$$\frac{t}{5} - 2 = \ln\frac{s}{100}$$

$$e^{t/5-2} = \frac{s}{100}$$

$$\frac{e^{t/5}}{e^2} = \frac{s}{100}$$

$$s = \left(13.53e^{t/5}\right) \text{ m}$$



When $s = 400$ m,

$$400 = 13.53e^{t/5}$$

$$t = 16.93 \text{ s} = 16.9 \text{ s} \qquad \textbf{Ans.}$$

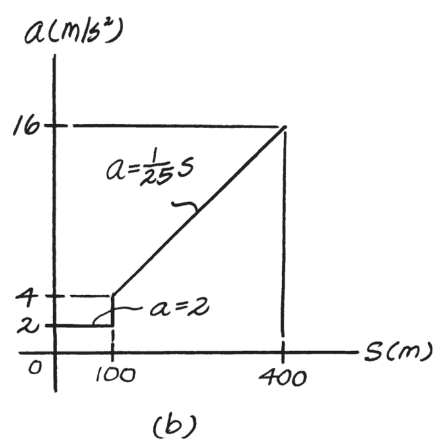
The s–t graph is shown in Fig. a.

$a-s$ Graph: For $0 \text{ m} \leq s < 100$ m,

$$a = v\frac{dv}{ds} = \left(2s^{1/2}\right)\left(s^{-1/2}\right) = 2 \text{ m/s}^2$$

For $100 \text{ m} < s \leq 400$ m,

$$a = v\frac{dv}{ds} = (0.2s)(0.2) = 0.04s$$



When $s = 100$ m and 400 m,

$$a\big|_{s=100\text{ m}} = 0.04(100) = 4 \text{ m/s}^2$$

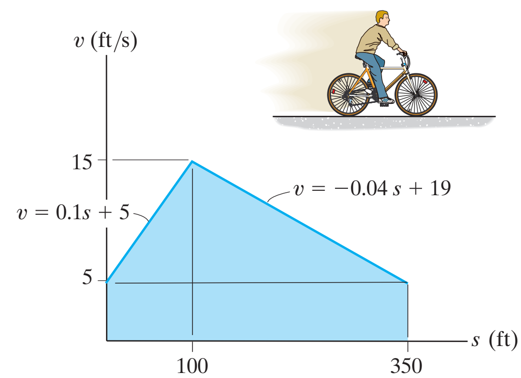
$$a\big|_{s=400\text{ m}} = 0.04(400) = 16 \text{ m/s}^2$$

The a–s graph is shown in Fig. b.

Ans:
When $s = 100$ m,
$t = 10$ s.
When $s = 400$ m,
$t = 16.9$ s.
$a\big|_{s=100\text{ m}} = 4 \text{ m/s}^2$
$a\big|_{s=400\text{ m}} = 16 \text{ m/s}^2$

12–67.

The $v-s$ graph of a cyclist traveling along a straight road is shown. Construct the $a-s$ graph.



SOLUTION

a–s Graph: For $0 \le s < 100$ ft,

$$\left(\overset{+}{\rightarrow}\right) \quad a = v\frac{dv}{ds} = \left(0.1s + 5\right)\left(0.1\right) = \left(0.01s + 0.5\right)\text{ ft/s}^2$$

Thus at $s = 0$ and 100 ft

$$a|_{s=0} = 0.01\left(0\right) + 0.5 = 0.5\text{ ft/s}^2$$

$$a|_{s=100\text{ ft}} = 0.01\left(100\right) + 0.5 = 1.5\text{ ft/s}^2$$

For 100 ft $< s \le 350$ ft,

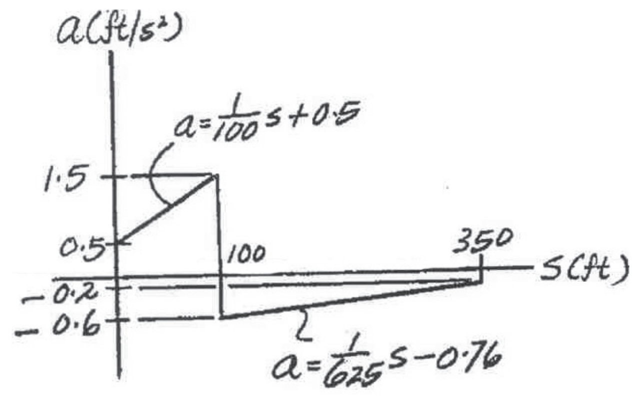
$$\left(\overset{+}{\rightarrow}\right) \quad a = v\frac{dv}{ds} = \left(-0.04s + 19\right)\left(-0.04\right) = \left(0.0016s - 0.76\right)\text{ ft/s}^2$$

Thus at $s = 100$ ft and 350 ft

$$a|_{s=100\text{ ft}} = 0.0016\left(100\right) - 0.76 = -0.6\text{ ft/s}^2$$

$$a|_{s=350\text{ ft}} = 0.0016\left(350\right) - 0.76 = -0.2\text{ ft/s}^2$$

The $a-s$ graph is shown in Fig. a.



(a)

Thus at $s = 0$ and 100 ft

$$a|_{s=0} = 0.01\left(0\right) + 0.5 = 0.5\text{ ft/s}^2$$

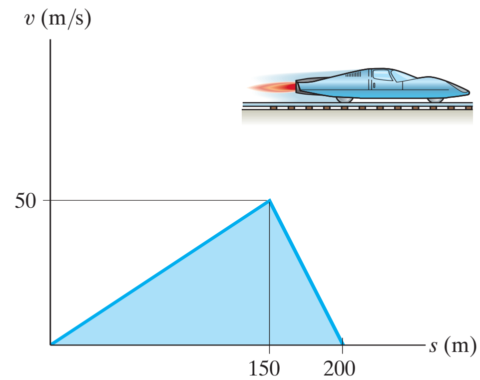
$$a|_{s=100\text{ ft}} = 0.01\left(100\right) + 0.5 = 1.5\text{ ft/s}^2$$

At $s = 100$ ft, a changes from $a_{max} = 1.5\text{ ft/s}^2$ to $a_{min} = -0.6\text{ ft/s}^2$.

Ans:
At $s = 100$ s,
a changes from $a_{max} = 1.5\text{ ft/s}^2$
to $a_{min} = -0.6\text{ ft/s}^2$.

***12–68.**

The v–s graph for a test vehicle is shown. Determine its acceleration when $s = 100$ m and when $s = 175$ m.



SOLUTION

$0 \le s \le 150\text{m}$: $\quad \nu = \frac{1}{3}s,$

$$d\nu = \frac{1}{3}ds$$

$$\nu\, d\nu = a\, ds$$

$$\frac{1}{3}s\left(\frac{1}{3}ds\right) = a\, ds$$

$$a = \frac{1}{9}s$$

At $s = 100$ m, $\quad a = \frac{1}{9}(100) = 11.1 \text{ m/s}^2$ **Ans.**

$150 \le s \le 200$ m; $\quad \nu = 200 - s,$

$$d\nu = -ds$$

$$\nu\, d\nu = a\, ds$$

$$(200 - s)(-ds) = a\, ds$$

$$a = s - 200$$

At $s = 175$ m, $\quad a = 175 - 200 = -25 \text{ m/s}^2$ **Ans.**

Ans:
At $s = 100$ s, $\quad a = 11.1 \text{ m/s}^2$
At $s = 175$ m, $\quad a = -25 \text{ m/s}^2$

12–69.

If the velocity of a particle is defined as $\mathbf{v}(t) = \{0.8t^2\mathbf{i} + 12t^{1/2}\mathbf{j} + 5\mathbf{k}\}$ m/s, determine the magnitude and coordinate direction angles α, β, γ of the particle's acceleration when $t = 2$ s.

SOLUTION

$$\mathbf{v}(t) = 0.8t^2\,\mathbf{i} + 12t^{1/2}\,\mathbf{j} + 5\mathbf{k}$$

$$\mathbf{a} = \frac{dv}{dt} = 1.6\mathbf{i} + 6t^{1/2}\mathbf{j}$$

When $t = 2$ s, $\quad \mathbf{a} = 3.2\mathbf{i} + 4.243\mathbf{j}$

$$a = \sqrt{(3.2)^2 + (4.243)^2} = 5.31 \text{ m/s}^2$$ **Ans.**

$$u_o = \frac{\mathbf{a}}{a} = 0.6022\mathbf{i} + 0.7984\mathbf{j}$$

$$\alpha = \cos^{-1}(0.6022) = 53.0^\circ$$ **Ans.**

$$\beta = \cos^{-1}(0.7984) = 37.0^\circ$$ **Ans.**

$$\gamma = \cos^{-1}(0) = 90.0^\circ$$ **Ans.**

Ans:
$a = 5.31 \text{ m/s}^2$
$\alpha = 53.0^\circ$
$\beta = 37.0^\circ$
$\gamma = 90.0^\circ$

12–70.

The velocity of a particle is $\mathbf{v} = \{3\mathbf{i} + (6 - 2t)\mathbf{j}\}$ m/s, where t is in seconds. If $\mathbf{r} = \mathbf{0}$ when $t = 0$, determine the displacement of the particle during the time interval $t = 1$ s to $t = 3$ s.

SOLUTION

Position: The position $\mathbf{r}$ of the particle can be determined by integrating the kinematic equation $d\mathbf{r} = \mathbf{v}dt$ using the initial condition $\mathbf{r} = \mathbf{0}$ at $t = 0$ as the integration limit. Thus,

$$d\mathbf{r} = \mathbf{v}dt$$

$$\int_0^{\mathbf{r}} d\mathbf{r} = \int_0^t \left[3\mathbf{i} + (6 - 2t)\mathbf{j}\right]dt$$

$$\mathbf{r} = \left[3t\mathbf{i} + \left(6t - t^2\right)\mathbf{j}\right]\text{m}$$

When $t = 1$ s and 3 s,

$$r\big|_{t=1\text{ s}} = 3(1)\mathbf{i} + \left[6(1) - 1^2\right]\mathbf{j} = [3\mathbf{i} + 5\mathbf{j}]\text{ m/s}$$

$$r\big|_{t=3\text{ s}} = 3(3)\mathbf{i} + \left[6(3) - 3^2\right]\mathbf{j} = [9\mathbf{i} + 9\mathbf{j}]\text{ m/s}$$

Thus, the displacement of the particle is

$$\Delta\mathbf{r} = \mathbf{r}\big|_{t=3\text{ s}} - \mathbf{r}\big|_{t=1\text{ s}}$$

$$= (9\mathbf{i} + 9\mathbf{j}) - (3\mathbf{i} + 5\mathbf{j})$$

$$= \{6\mathbf{i} + 4\mathbf{j}\}\text{ m}$$ **Ans.**

Ans:
$\Delta\mathbf{r} = \{6\mathbf{i} + 4\mathbf{j}\}$ m

12–71.

A particle, originally at rest and located at point (3 ft, 2 ft, 5 ft), is subjected to an acceleration of $\mathbf{a} = \{6t\mathbf{i} + 12t^2\mathbf{k}\}$ ft/s^2. Determine the particle's position (x, y, z) at $t = 1$ s.

SOLUTION

Velocity: The velocity expressed in Cartesian vector form can be obtained by applying Eq. 12–9.

$$dv = \text{a}dt$$

$$\int_0^v dv = \int_0^t (6t\mathbf{i} + 12t^2\mathbf{k})\, dt$$

$$v = \{3t^2\mathbf{i} + 4t^3\mathbf{k}\} \text{ ft/s}$$

Position: The position expressed in Cartesian vector form can be obtained by applying Eq. 12–7.

$$dr = \text{v}dt$$

$$\int_{\text{r}_1}^{\text{r}} dr = \int_0^t (3t^2\mathbf{i} + 4t^3\mathbf{k})\, dt$$

$$\text{r} - (3\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}) = t^3\mathbf{i} + t^4\mathbf{k}$$

$$\text{r} = \{(t^3 + 3)\,\mathbf{i} + 2\mathbf{j} + (t^4 + 5)\mathbf{k}\} \text{ ft}$$

When $t = 1\ s$, $\text{r} = (1^3 + 3)\mathbf{i} + 2\mathbf{j} + (1^4 + 5)\mathbf{k} = \{4\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}\}$ ft.

The coordinates of the particle are

(4 ft, 2 ft, 6 ft) **Ans.**

Ans:
(4 ft, 2 ft, 6 ft)

***12–72.**

The velocity of a particle is given by $\mathbf{v} = \{16t^2\mathbf{i} + 4t^3\mathbf{j} + (5t + 2)\mathbf{k}\}$ m/s, where t is in seconds. If the particle is at the origin when $t = 0$, determine the magnitude of the particle's acceleration when $t = 2$ s. Also, what is the x, y, z coordinate position of the particle at this instant?

SOLUTION

Acceleration: The acceleration expressed in Cartesian vector form can be obtained by applying Eq. 12–9.

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \{32t\mathbf{i} + 12t^2\mathbf{j} + 5\mathbf{k}\} \text{ m/s}^2$$

When $t = 2$ s, $\mathbf{a} = 32(2)\mathbf{i} + 12(2^2)\mathbf{j} + 5\mathbf{k} = \{64\mathbf{i} + 48\mathbf{j} + 5\mathbf{k}\}$ m/s^2. The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{64^2 + 48^2 + 5^2} = 80.2 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Position: The position expressed in Cartesian vector form can be obtained by applying Eq. 12–7.

$$d\mathbf{r} = \mathbf{v}\,dt$$

$$\int_0^{\mathbf{r}} d\mathbf{r} = \int_0^t \left(16t^2\mathbf{i} + 4t^3\mathbf{j} + (5t + 2)\mathbf{k}\right) dt$$

$$\mathbf{r} = \left[\frac{16}{3}t^3\mathbf{i} + t^4\mathbf{j} + \left(\frac{5}{2}t^2 + 2t\right)\mathbf{k}\right] \text{ m}$$

When $t = 2$ s,

$$\mathbf{r} = \frac{16}{3}\left(2^3\right)\mathbf{i} + \left(2^4\right)\mathbf{j} + \left[\frac{5}{2}\left(2^2\right) + 2(2)\right]\mathbf{k} = \{42.7\mathbf{i} + 16.0\mathbf{j} + 14.0\mathbf{k}\} \text{ m.}$$

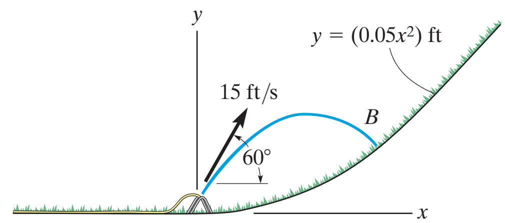
Thus, the coordinate of the particle is

$$(42.7, 16.0, 14.0) \text{ m} \qquad \textbf{Ans.}$$

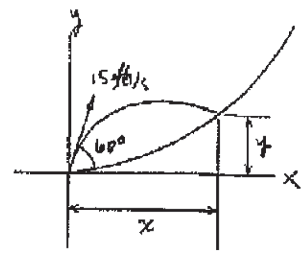
Ans:
(42.7, 16.0, 14.0) m

12–73.

The water sprinkler, positioned at the base of a hill, releases a stream of water with a velocity of 15 ft/s as shown. Determine the point $B(x, y)$ where the water strikes the ground on the hill. Assume that the hill is defined by the equation $y = (0.05x^2)$ ft and neglect the size of the sprinkler.



SOLUTION



$v_x = 15 \cos 60° = 7.5 \text{ ft/s} \qquad v_y = 15 \sin 60° = 12.99 \text{ ft/s}$

$(\overset{+}{\rightarrow}) \quad s = v_0 t$

$x = 7.5t$

$(+\uparrow) \quad s = s_o + v_o t + \frac{1}{2} a_c t^2$

$y = 0 + 12.99t + \frac{1}{2}(-32.2)t^2$

$y = 1.732x - 0.286x^2$

Since $y = 0.05x^2$,

$0.05x^2 = 1.732x - 0.286x^2$

$x(0.336x - 1.732) = 0$

$x = 5.15 \text{ ft}$ **Ans.**

$y = 0.05(5.15)^2 = 1.33 \text{ ft}$ **Ans.**

Also,

$(\overset{+}{\rightarrow}) \quad s = v_0 t$

$x = 15 \cos 60° t$

$(+\uparrow) \quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$y = 0 + 15 \sin 60° t + \frac{1}{2}(-32.2)t^2$

Since $y = 0.05x^2$

$12.99t - 16.1t^2 = 2.8125t^2 \quad t = 0.6869 \text{ s}$

So that,

$x = 15 \cos 60° \,(0.6868) = 5.15 \text{ ft}$ **Ans.**

$y = 0.05(5.15)^2 = 1.33 \text{ ft}$ **Ans.**

Ans:
(5.15 ft, 1.33 ft)

12–74.

A particle, originally at rest and located at point (3 ft, 2 ft, 5 ft), is subjected to an acceleration $\mathbf{a} = \{6t\mathbf{i} + 12t^2\mathbf{k}\}$ ft/s^2. Determine the particle's position (x, y, z) when $t = 2$ s.

SOLUTION

$\mathbf{a} = 6t\mathbf{i} + 12t^2\mathbf{k}$

$$\int_0^v d\mathbf{v} = \int_0^t (6t\mathbf{i} + 12t^2\mathbf{k})\, dt$$

$\mathbf{v} = 3t^2\mathbf{i} + 4t^3\mathbf{k}$

$$\int_{\mathrm{r}_0}^{\mathrm{r}} d\mathbf{r} = \int_0^t (3t^2\mathbf{i} + 4t^3\mathbf{k})\, dt$$

$\mathbf{r} - (3\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}) = t^3\mathbf{i} + t^4\mathbf{k}$

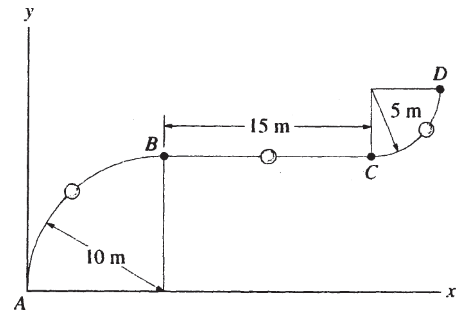
When $t = 2$ s

$\mathbf{r} = \{11\mathbf{i} + 2\mathbf{j} + 21\mathbf{k}\}$ ft **Ans.**

Ans:
$\mathbf{r} = \{11\mathbf{i} + 2\mathbf{j} + 21\mathbf{k}\}$ ft

12–75.

A particle travels along the curve from A to B in 2 s. It takes 4 s for it to go from B to C and then 3 s to go from C to D. Determine its average speed when it goes from A to D.



SOLUTION

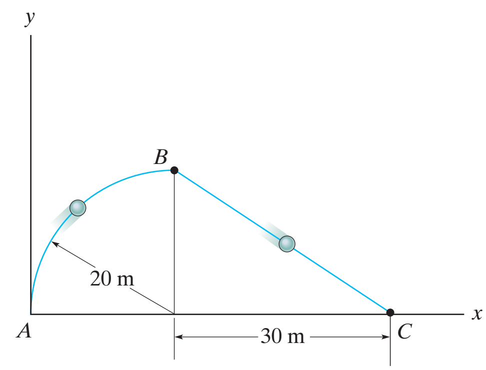
$$s_T = \frac{1}{4}(2\pi)(10)) + 15 + \frac{1}{4}(2\pi(5)) = 38.56$$

$$v_{sp} = \frac{s_T}{t_t} = \frac{38.56}{2 + 4 + 3} = 4.28 \text{ m/s}$$ **Ans.**

Ans:
$(v_{\text{sp}})_{\text{avg}} = 4.28 \text{ m/s}$

***12–76.**

A particle travels along the curve from A to B in 5 s. It takes 8 s for it to go from B to C and then 10 s to go from C to A. Determine its average speed when it goes around the closed path.



SOLUTION

The total distance traveled is

$$S_{\text{Tot}} = S_{AB} + S_{BC} + S_{CA}$$

$$= 20\left(\frac{\pi}{2}\right) + \sqrt{20^2 + 30^2} + (30 + 20)$$

$$= 117.47 \text{ m}$$

The total time taken is

$$t_{\text{Tot}} = t_{AB} + t_{BC} + t_{CA}$$

$$= 5 + 8 + 10$$

$$= 23 \text{ s}$$

Thus, the average speed is

$$(v_{sp})_{avg} = \frac{S_{\text{Tot}}}{t_{\text{Tot}}} = \frac{117.47 \text{ m}}{23 \text{ s}} = 5.107 \text{ m/s} = 5.11 \text{ m/s}$$ **Ans.**

Ans:
$(v_{sp})_{avg} = 5.11 \text{ m/s}$

12–77.

The position of a crate sliding down a ramp is given by $x = (0.25t^3)$ m, $y = (1.5t^2)$ m, $z = (6 - 0.75t^{5/2})$ m, where t is in seconds. Determine the magnitude of the crate's velocity and acceleration when $t = 2$ s.

SOLUTION

Velocity: By taking the time derivative of x, y, and z, we obtain the x, y, and z components of the crate's velocity.

$$v_x = \dot{x} = \frac{d}{dt}\left(0.25t^3\right) = \left(0.75t^2\right) \text{ m/s}$$

$$v_y = \dot{y} = \frac{d}{dt}\left(1.5t^2\right) = \left(3t\right) \text{ m/s}$$

$$v_z = \dot{z} = \frac{d}{dt}\left(6 - 0.75t^{5/2}\right) = \left(-1.875t^{3/2}\right) \text{ m/s}$$

When $t = 2$ s,

$$v_x = 0.75\left(2^2\right) = 3 \text{ m/s} \qquad v_y = 3(2) = 6 \text{ m/s} \qquad v_z = -1.875\left(2\right)^{3/2} = -5.303 \text{ m/s}$$

Thus, the magnitude of the crate's velocity is

$$v = \sqrt{v_x{}^2 + v_y{}^2 + v_z{}^2} = \sqrt{3^2 + 6^2 + (-5.303)^2} = 8.551 \text{ ft/s} = 8.55 \text{ ft} \qquad \textbf{Ans.}$$

Acceleration: The x, y, and z components of the crate's acceleration can be obtained by taking the time derivative of the results of v_x, v_y, and v_z, respectively.

$$a_x = \dot{v}_x = \frac{d}{dt}\left(0.75t^2\right) = (1.5t) \text{ m/s}^2$$

$$a_y = \dot{v}_y = \frac{d}{dt}(3t) = 3 \text{ m/s}^2$$

$$a_z = \dot{v}_z = \frac{d}{dt}\left(-1.875t^{3/2}\right) = \left(-2.815t^{1/2}\right) \text{ m/s}^2$$

When $t = 2$ s,

$$a_x = 1.5(2) = 3 \text{ m/s}^2 \qquad a_y = 3 \text{ m/s}^2 \qquad a_z = -2.8125\left(2^{1/2}\right) = -3.977 \text{ m/s}^2$$

Thus, the magnitude of the crate's acceleration is

$$a = \sqrt{a_x{}^2 + a_y{}^2 + a_z{}^2} = \sqrt{3^2 + 3^2 + (-3.977)^2} = 5.815 \text{ m/s}^2 = 5.82 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v = 8.55$ ft/s
$a = 5.82$ m/s^2

12–78.

A rocket is fired from rest at $x = 0$ and travels along a parabolic trajectory described by $y^2 = [120(10^3)x]$ m. If the x component of acceleration is $a_x = \left(\frac{1}{4}t^2\right)$ m/s², where t is in seconds, determine the magnitude of the rocket's velocity and acceleration when $t = 10$ s.

SOLUTION

Position: The parameter equation of x can be determined by integrating a_x twice with respect to t.

$$\int dv_x = \int a_x dt$$

$$\int_0^{v_x} dv_x = \int_0^t \frac{1}{4}t^2 dt$$

$$v_x = \left(\frac{1}{12}t^3\right) \text{ m/s}$$

$$\int dx = \int v_x dt$$

$$\int_0^x dx = \int_0^t \frac{1}{12}t^3 dt$$

$$x = \left(\frac{1}{48}t^4\right) \text{ m}$$

Substituting the result of x into the equation of the path,

$$y^2 = 120\left(10^3\right)\left(\frac{1}{48}t^4\right)$$

$$y = \left(50t^2\right) \text{ m}$$

Velocity:

$$v_y = \dot{y} = \frac{d}{dt}\left(50t^2\right) = \left(100t\right) \text{ m/s}$$

When $t = 10$ s,

$$v_x = \frac{1}{12}\left(10^3\right) = 83.33 \text{ m/s} \qquad v_y = 100(10) = 1000 \text{ m/s}$$

Thus, the magnitude of the rocket's velocity is

$$v = \sqrt{v_x{}^2 + v_y{}^2} = \sqrt{83.33^2 + 1000^2} = 1003 \text{ m/s}$$ **Ans.**

Acceleration:

$$a_y = \dot{v}_y = \frac{d}{dt}(100t) = 100 \text{ m/s}^2$$

When $t = 10$ s,

$$a_x = \frac{1}{4}\left(10^2\right) = 25 \text{ m/s}^2$$

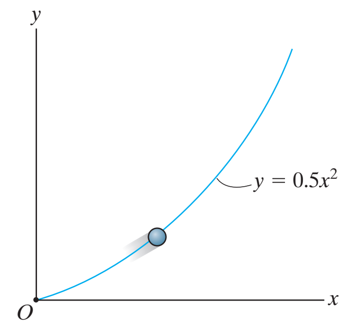
Thus, the magnitude of the rocket's acceleration is

$$a = \sqrt{a_x{}^2 + a_y{}^2} = \sqrt{25^2 + 100^2} = 103 \text{ m/s}^2$$ **Ans.**

Ans:
$v = 1003$ m/s
$a = 103$ m/s²

12–79.

The particle travels along the path defined by the parabola $y = 0.5x^2$. If the component of velocity along the x axis is $v_x = (5t)$ ft/s, where t is in seconds, determine the particle's distance from the origin O and the magnitude of its acceleration when $t = 1$ s. When $t = 0$, $x = 0$, $y = 0$.



SOLUTION

Position: The x position of the particle can be obtained by applying the $v_x = \frac{dx}{dt}$.

$$dx = v_x dt$$

$$\int_0^x dx = \int_0^t 5t dt$$

$$x = \left(2.50t^2\right) \text{ ft}$$

Thus, $y = 0.5\left(2.50t^2\right)^2 = \left(3.125t^4\right)$ ft. At $t = 1$ s, $x = 2.5\left(1^2\right) = 2.50$ ft and $y = 3.125\left(1^4\right) = 3.125$ ft. The particle's distance from the origin at this moment is

$$d = \sqrt{(2.50 - 0)^2 + (3.125 - 0)^2} = 4.00 \text{ ft}$$ **Ans.**

Acceleration: Taking the first derivative of the path $y = 0.5x^2$, we have $\dot{y} = x\dot{x}$. The second derivative of the path gives

$$\ddot{y} = \dot{x}^2 + x\ddot{x} \qquad \textbf{(1)}$$

However, $\dot{x} = v_x$, $\ddot{x} = a_x$ and $\ddot{y} = a_y$. Thus, Eq. (1) becomes

$$a_y = v_x^2 + xa_x \qquad \textbf{(2)}$$

When $t = 1$ s, $v_x = 5(1) = 5$ ft/s $a_x = \frac{dv_x}{dt} = 5$ ft/s^2, and $x = 2.50$ ft. Then, from Eq. (2)

$$a_y = 5^2 + 2.50(5) = 37.5 \text{ ft/s}^2$$

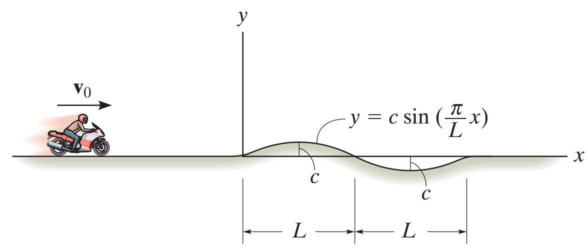
Also,

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{5^2 + 37.5^2} = 37.8 \text{ ft/s}^2$$ **Ans.**

Ans:
$d = 4.00$ ft
$a = 37.8$ ft/s^2

***12–80.**

The motorcycle travels with constant speed v_0 along the path that, for a short distance, takes the form of a sine curve. Determine the x and y components of its velocity at any instant on the curve.



SOLUTION

$$y = c \sin\left(\frac{\pi}{L}x\right)$$

$$\dot{y} = \frac{\pi}{L}c\left(\cos\frac{\pi}{L}x\right)\dot{x}$$

$$v_y = \frac{\pi}{L}c\, v_x\left(\cos\frac{\pi}{L}x\right)$$

$$v_0^2 = v_y^2 + v_x^2$$

$$v_0^2 = v_x^2\left[1 + \left(\frac{\pi}{L}c\right)^2 \cos^2\left(\frac{\pi}{L}x\right)\right]$$

$$v_x = v_0\left[1 + \left(\frac{\pi}{L}c\right)^2 \cos^2\left(\frac{\pi}{L}x\right)\right]^{-\frac{1}{2}}$$ **Ans.**

$$v_y = \frac{v_0\,\pi c}{L}\left(\cos\frac{\pi}{L}x\right)\left[1 + \left(\frac{\pi}{L}c\right)^2 \cos^2\left(\frac{\pi}{L}x\right)\right]^{-\frac{1}{2}}$$ **Ans.**

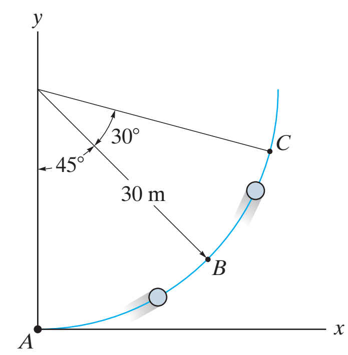
Ans:

$$v_x = v_0\left[1 + \left(\frac{\pi}{L}c\right)^2 \cos^2\left(\frac{\pi}{L}x\right)\right]^{-\frac{1}{2}}$$

$$v_y = \frac{v_0\,\pi c}{L}\left(\cos\frac{\pi}{L}x\right)\left[1 + \left(\frac{\pi}{L}c\right)^2 \cos^2\left(\frac{\pi}{L}x\right)\right]^{-\frac{1}{2}}$$

12–81.

A particle travels along the circular path from A to B in 1 s. If it takes 3 s for it to go from A to C, determine its *average velocity* when it goes from B to C.



SOLUTION

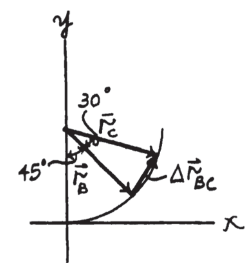
Position: The coordinates for points B and C are [30 sin 45°, 30 − 30 cos 45°] and [30 sin 75°, 30 − 30 cos 75°]. Thus,

$$\mathbf{r}_B = (30 \sin 45° - 0)\mathbf{i} + [(30 - 30 \cos 45°) - 30]\mathbf{j}$$

$$= \{21.21\mathbf{i} - 21.21\mathbf{j}\} \text{ m}$$

$$\mathbf{r}_C = (30 \sin 75° - 0)\mathbf{i} + [(30 - 30 \cos 75°) - 30]\mathbf{j}$$

$$= \{28.98\mathbf{i} - 7.765\mathbf{j}\} \text{ m}$$



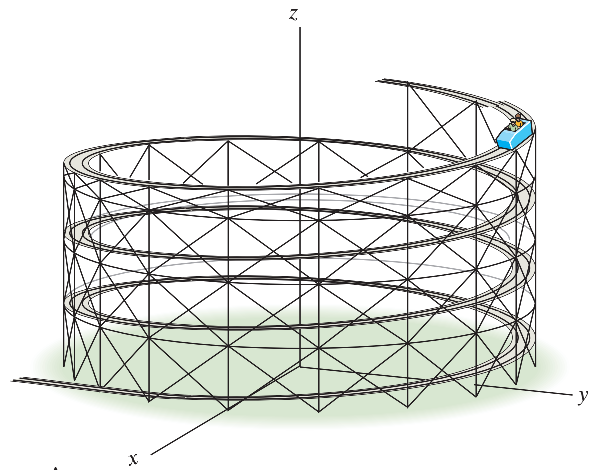
Average Velocity: The displacement from point B to C is $\Delta\mathbf{r}_{BC} = \mathbf{r}_C - \mathbf{r}_B = (28.98\mathbf{i} - 7.765\mathbf{j}) - (21.21\mathbf{i} - 21.21\mathbf{j}) = \{7.765\mathbf{i} + 13.45\mathbf{j}\}$ m.

$$(\mathbf{v}_{BC})_{\text{avg}} = \frac{\Delta\mathbf{r}_{BC}}{\Delta t} = \frac{7.765\mathbf{i} + 13.45\mathbf{j}}{3 - 1} = \{3.88\mathbf{i} + 6.72\mathbf{j}\} \text{ m/s}$$ **Ans.**

Ans:
$(\mathbf{v}_{BC})_{\text{avg}} = \{3.88\mathbf{i} + 6.72\mathbf{j}\}$ m/s

12–82.

The roller coaster car travels down the helical path at constant speed such that the parametric equations that define its position are $x = c \sin kt$, $y = c \cos kt$, $z = h - bt$, where c, h, and b are constants. Determine the magnitudes of its velocity and acceleration.



SOLUTION

$x = c \sin kt \qquad \dot{x} = \text{ck} \cos kt \qquad \ddot{x} = -ck^2 \sin kt$

$y = c \cos kt \qquad \dot{y} = -\text{ck} \sin kt \qquad \ddot{y} = -ck^2 \cos kt$

$z = h - bt \qquad \dot{z} = -b \qquad \ddot{z} = 0$

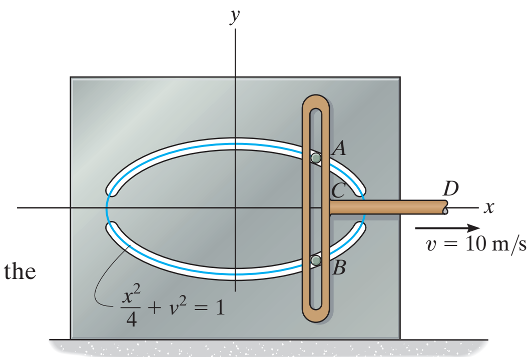
$v = \sqrt{(ck \cos kt)^2 + (-ck \sin kt)^2 + (-b)^2} = \sqrt{c^2k^2 + b^2}$ **Ans.**

$a = \sqrt{(-ck^2 \sin kt)^2 + (-ck^2 \cos kt)^2 + 0} = ck^2$ **Ans.**

Ans:
$v = \sqrt{c^2 k^2 + b^2}$
$a = ck^2$

12–83.

Pegs A and B are restricted to move in the elliptical slots due to the motion of the slotted link. If the link moves with a constant speed of 10 m/s, determine the magnitude of the velocity and acceleration of peg A when $x = 1$ m.



SOLUTION

Velocity: The x and y components of the peg's velocity can be related by taking the first time derivative of the path's equation.

$$\frac{x^2}{4} + y^2 = 1$$

$$\frac{1}{4}(2x\dot{x}) + 2y\dot{y} = 0$$

$$\frac{1}{2}x\dot{x} + 2y\dot{y} = 0$$

or

$$\frac{1}{2}xv_x + 2yv_y = 0 \qquad \textbf{(1)}$$

At $x = 1$ m,

$$\frac{(1)^2}{4} + y^2 = 1 \qquad y = \frac{\sqrt{3}}{2}\text{ m}$$

Here, $v_x = 10$ m/s and $x = 1$. Substituting these values into Eq. (1),

$$\frac{1}{2}(1)(10) + 2\left(\frac{\sqrt{3}}{2}\right)v_y = 0 \qquad v_y = -2.887\text{ m/s} = 2.887\text{ m/s} \downarrow$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{v_x{}^2 + v_y{}^2} = \sqrt{10^2 + 2.887^2} = 10.4\text{ m/s} \qquad \textbf{Ans.}$$

Acceleration: The x and y components of the peg's acceleration can be related by taking the second time derivative of the path's equation.

$$\frac{1}{2}(\dot{x}\dot{x} + x\ddot{x}) + 2(\dot{y}\dot{y} + y\ddot{y}) = 0$$

$$\frac{1}{2}\left(\dot{x}^2 + x\ddot{x}\right) + 2\left(\dot{y}^2 + y\ddot{y}\right) = 0$$

or

$$\frac{1}{2}\left(v_x{}^2 + xa_x\right) + 2\left(v_y{}^2 + ya_y\right) = 0 \qquad \textbf{(2)}$$

Since v_x is constant, $a_x = 0$. When $x = 1$ m, $y = \frac{\sqrt{3}}{2}$ m, $v_x = 10$ m/s, and $v_y = -2.887$ m/s. Substituting these values into Eq. (2),

$$\frac{1}{2}\left(10^2 + 0\right) + 2\left[(-2.887)^2 + \frac{\sqrt{3}}{2}a_y\right] = 0$$

$$a_y = -38.49\text{ m/s}^2 = 38.49\text{ m/s}^2 \downarrow$$

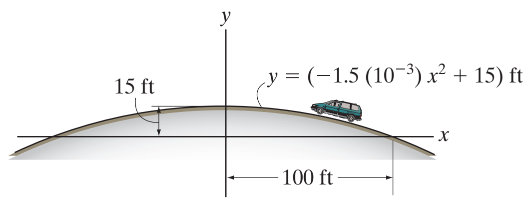
Thus, the magnitude of the peg's acceleration is

$$a = \sqrt{a_x{}^2 + a_y{}^2} = \sqrt{0^2 + (-38.49)^2} = 38.5\text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v = 10.4$ m/s
$a = 38.5$ m/s^2

***12–84.**

The van travels over the hill described by $y = (-1.5(10^{-3})\,x^2 + 15)$ ft. If it has a constant speed of 75 ft/s, determine the x and y components of the van's velocity and acceleration when $x = 50$ ft.



SOLUTION

Velocity: The x and y components of the van's velocity can be related by taking the first time derivative of the path's equation using the chain rule.

$$y = -1.5\left(10^{-3}\right)x^2 + 15$$

$$\dot{y} = -3\left(10^{-3}\right)x\dot{x}$$

or

$$v_y = -3\left(10^{-3}\right)xv_x$$

When $x = 50$ ft,

$$v_y = -3\left(10^{-3}\right)(50)v_x = -0.15v_x \qquad \textbf{(1)}$$

The magnitude of the van's velocity is

$$v = \sqrt{{v_x}^2 + {v_y}^2} \qquad \textbf{(2)}$$

Substituting $v = 75$ ft/s and Eq. (1) into Eq. (2),

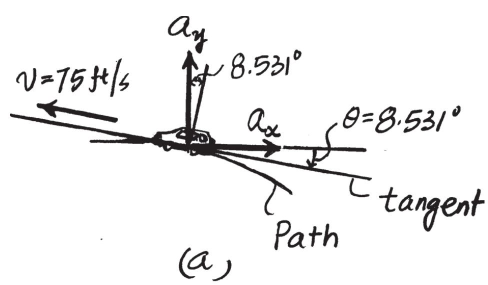
$$75 = \sqrt{{v_x}^2 + (-0.15v_x)^2}$$

$$v_x = 74.2 \text{ ft/s} \leftarrow \qquad \textbf{Ans.}$$

Substituting the result of v_x into Eq. (1), we obtain

$$v_y = -0.15(-74.17) = 11.12 \text{ ft/s} = 11.1 \text{ ft/s} \uparrow \qquad \textbf{Ans.}$$

Acceleration: The x and y components of the van's acceleration can be related by taking the second time derivative of the path's equation using the chain rule.



$$\ddot{y} = -3\left(10^{-3}\right)(\dot{x}\dot{x} + x\ddot{x})$$

or

$$a_y = -3\left(10^{-3}\right)\left({v_x}^2 + xa_x\right)$$

When $x = 50$ ft, $v_x = -74.17$ ft/s. Thus,

$$a_y = -3\left(10^{-3}\right)\left[(-74.17)^2 + 50a_x\right]$$

$$a_y = -(16.504 + 0.15a_x) \qquad \textbf{(3)}$$

Since the van travels with a constant speed along the path, its acceleration along the tangent of the path is equal to zero. Here, the angle that the tangent makes with the horizontal at $x = 50$ ft is $\theta = \tan^{-1}\left(\frac{dy}{dx}\right)\Big|_{x=50\text{ ft}} = \tan^{-1}\left[-3\left(10^{-3}\right)x\right]\Big|_{x=50\text{ ft}} = \tan^{-1}(-0.15) = -8.531°$.
Thus, from the diagram shown in Fig. *a*,

$$a_x \cos 8.531° - a_y \sin 8.531° = 0 \qquad \textbf{(4)}$$

Solving Eqs. (3) and (4) yields

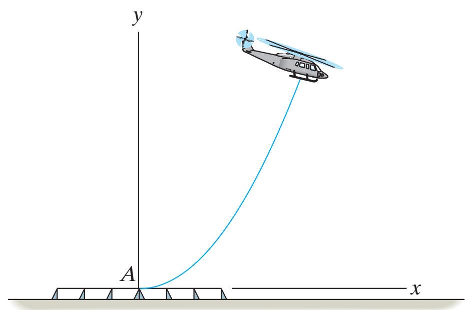
$$a_x = -2.42 \text{ ft/s} = 2.42 \text{ ft/s}^2 \leftarrow \qquad \textbf{Ans.}$$

$$a_y = -16.1 \text{ ft/s} = 16.1 \text{ ft/s}^2 \downarrow \qquad \textbf{Ans.}$$

Ans:
$v_x = 74.2$ ft/s $\leftarrow$
$v_y = 11.1$ ft/s $\uparrow$
$a_x = 2.42$ ft/s^2 $\leftarrow$
$a_y = 16.1$ ft/s^2 $\downarrow$

12–85.

The flight path of the helicopter as it takes off from A is defined by the parametric equations $x = (2t^2)$ m and $y = (0.04t^3)$ m, where t is the time in seconds. Determine the distance the helicopter is from point A and the magnitudes of its velocity and acceleration when $t = 10$ s.



SOLUTION

$x = 2t^2 \qquad y = 0.04t^3$

At $t = 10$ s, $\quad x = 200$ m $\quad y = 40$ m

$d = \sqrt{(200)^2 + (40)^2} = 204$ m **Ans.**

$$v_x = \frac{dx}{dt} = 4t$$

$$a_x = \frac{dv_x}{dt} = 4$$

$$v_y = \frac{dy}{dt} = 0.12t^2$$

$$a_y = \frac{dv_y}{dt} = 0.24t$$

At $t = 10$ s,

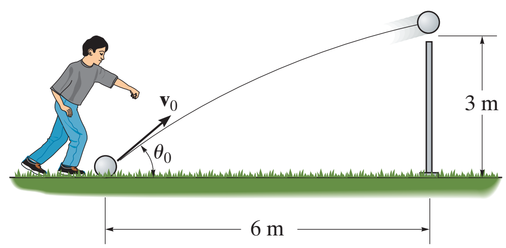
$v = \sqrt{(40)^2 + (12)^2} = 41.8$ m/s **Ans.**

$a = \sqrt{(4)^2 + (2.4)^2} = 4.66$ m/s^2 **Ans.**

Ans:
$d = 204$ m
$v = 41.8$ m/s
$a = 4.66$ m/s^2

12–86.

Determine the minimum initial velocity v_0 and the corresponding angle θ_0 at which the ball must be kicked in order for it to just cross over the 3-m high fence.



SOLUTION

Coordinate System: The $x-y$ coordinate system will be set so that its origin coincides with the ball's initial position.

x-Motion: Here, $(v_0)_x = v_0 \cos\theta$, $x_0 = 0$, and $x = 6$ m. Thus,

$$(\overset{+}{\rightarrow}) \qquad x = x_0 + (v_0)x^t$$

$$6 = 0 + (v_0 \cos\theta)t$$

$$t = \frac{6}{v_0 \cos\theta} \qquad \textbf{(1)}$$

y-Motion: Here, $(v_0)_x = v_0 \sin\theta$, $a_y = -g = -9.81\,\text{m/s}^2$, and $y_0 = 0$. Thus,

$$(+\uparrow) \qquad y = y_0 + (v_0)_y t + \frac{1}{2}a_y t^2$$

$$3 = 0 + v_0 (\sin\theta)\, t + \frac{1}{2}(-9.81)t^2$$

$$3 = v_0 (\sin\theta)\, t - 4.905t^2 \qquad \textbf{(2)}$$

Substituting Eq. (1) into Eq. (2) yields

$$v_0 = \sqrt{\frac{58.86}{\sin 2\theta - \cos^2\theta}} \qquad \textbf{(3)}$$

From Eq. (3), we notice that v_0 is minimum when $f(\theta) = \sin 2\theta - \cos^2\theta$ is maximum. This requires $\frac{df(\theta)}{d\theta} = 0$

$$\frac{df(\theta)}{d\theta} = 2\cos 2\theta + \sin 2\theta = 0$$

$$\tan 2\theta = -2$$

$$2\theta = 116.57°$$

$$\theta = 58.28° = 58.3° \qquad \textbf{Ans.}$$

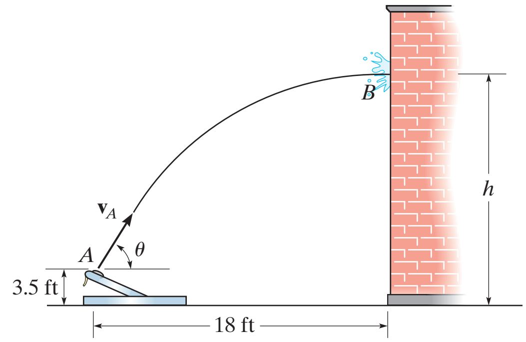
Substituting the result of θ into Eq. (2), we have

$$(v_0)_{min} = \sqrt{\frac{58.86}{\sin 116.57° - \cos^2 58.28°}} = 9.76 \text{ m/s} \qquad \textbf{Ans.}$$

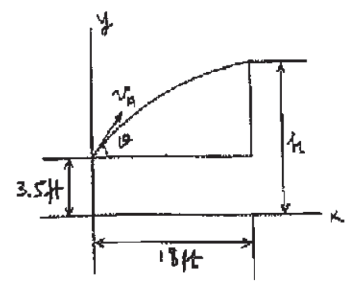
Ans:
$\theta = 58.3°$
$(v_0)_{\text{min}} = 9.76\,\text{m/s}$

12–87.

The catapult is used to launch a ball such that it strikes the wall of the building at the maximum height of its trajectory. If it takes 1.5 s to travel from A to B, determine the velocity $\mathbf{v}_A$ at which it was launched, the angle of release θ, and the height h.



SOLUTION



$(\overset{+}{\rightarrow})\quad s = v_0 t$

$$18 = v_A \cos\theta(1.5) \qquad \textbf{(1)}$$

$(+\uparrow)\quad v^2 = v_0^2 + 2a_c(s - s_0)$

$$0 = (v_A \sin\theta)^2 + 2(-32.2)(h - 3.5)$$

$(+\uparrow)\quad v = v_0 + a_c t$

$$0 = v_A \sin\theta - 32.2(1.5) \qquad \textbf{(2)}$$

To solve, first divide Eq. (2) by Eq. (1) to get θ. Then

$$\theta = 76.0° \qquad \textbf{Ans.}$$

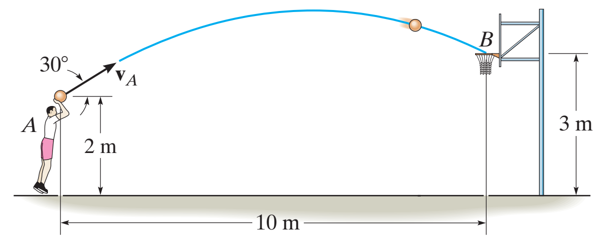
$$v_A = 49.8 \text{ ft/s} \qquad \textbf{Ans.}$$

$$h = 39.7 \text{ ft} \qquad \textbf{Ans.}$$

Ans:
$\theta = 76.0°$
$v_A = 49.8 \text{ ft/s}$
$h = 39.7 \text{ ft}$

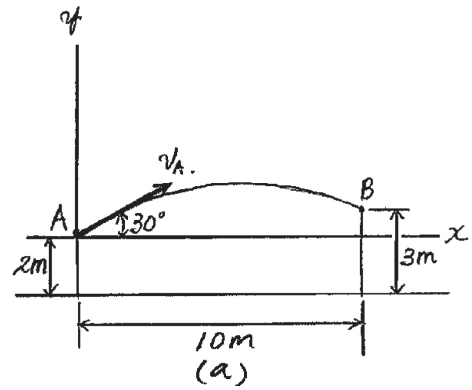
***12–88.**

Neglecting the size of the ball, determine the magnitude v_A of the basketball's initial velocity and its velocity when it passes through the basket.



SOLUTION

Coordinate System. The origin of the *x-y* coordinate system will be set to coinside with point *A* as shown in Fig. *a*



Horizontal Motion. Here $(v_A)_x = v_A \cos 30° \rightarrow$, $(s_A)_x = 0$ and $(s_B)_x = 10 \text{ m} \rightarrow$.

$$(\xrightarrow{+})\quad (s_B)_x = (s_A)_x + (v_A)_x t$$

$$10 = 0 + v_A \cos 30°\, t$$

$$t = \frac{10}{v_A \cos 30°} \qquad \textbf{(1)}$$

Also,

$$(\xrightarrow{+})\quad (v_B)_x = (v_A)_x = v_A \cos 30° \qquad \textbf{(2)}$$

Vertical Motion. Here, $(v_A)_y = v_A \sin 30° \uparrow$, $(s_A)_y = 0$, $(s_B)_y = 3 - 2 = 1 \text{ m}\uparrow$ and $a_y = 9.81 \text{ m/s}^2 \downarrow$

$$(+\uparrow)\quad (s_B)_y = (s_A)_y + (v_A)_y t + \frac{1}{2} a_y t^2$$

$$1 = 0 + v_A \sin 30°\, t + \frac{1}{2}(-9.81)t^2$$

$$4.905t^2 - 0.5\, v_A\, t + 1 = 0 \qquad \textbf{(3)}$$

Also

$$(+\uparrow)\quad (v_B)_y = (v_A)_y + a_y t$$

$$(v_B)_y = v_A \sin 30° + (-9.81)t$$

$$(v_B)_y = 0.5\, v_A - 9.81t \qquad \textbf{(4)}$$

Solving Eq. (1) and (3)

$$v_A = 11.705 \text{ m/s} = 11.7 \text{ m/s} \qquad \textbf{Ans.}$$

$$t = 0.9865 \text{ s}$$

Substitute these results into Eq. (2) and (4)

$$(v_B)_x = 11.705 \cos 30° = 10.14 \text{ m/s} \rightarrow$$

$$(v_B)_y = 0.5(11.705) - 9.81(0.9865) = -3.825 \text{ m/s} = 3.825 \text{ m/s} \downarrow$$

Thus, the magnitude of $\mathbf{v}_B$ is

$$v_B = \sqrt{(v_B)_x^2 + (v_B)_y^2} = \sqrt{10.14^2 + 3.825^2} = 10.83 \text{ m/s} = 10.8 \text{ m/s} \qquad \textbf{Ans.}$$

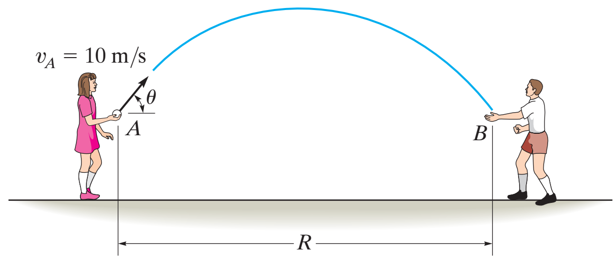
And its direction is defined by

$$\theta_B = \tan^{-1}\left[\frac{(v_B)_y}{(v_B)_x}\right] = \tan^{-1}\left(\frac{3.825}{10.14}\right) = 20.67° = 20.7° \qquad \textbf{Ans.}$$

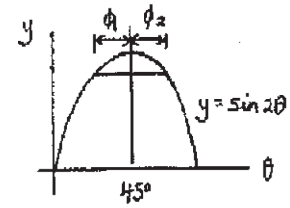
Ans:
$v_A = 11.7 \text{ m/s}$
$v_B = 10.8 \text{ m/s}$
$\theta = 20.7°$ ⦨

12–89.

The girl at A can throw a ball at $v_A = 10$ m/s. Calculate the maximum possible range $R = R_{max}$ and the associated angle θ at which it should be thrown. Assume the ball is caught at B at the same elevation from which it is thrown.



SOLUTION



$(\overset{+}{\rightarrow})$ $s = s_0 + v_0 t$

$$R = 0 + (10 \cos \theta)t$$

$(+\uparrow)$ $v = v_0 + a_c t$

$$-10 \sin \theta = 10 \sin \theta - 9.81t$$

$$t = \frac{20}{9.81} \sin \theta$$

Thus, $R = \dfrac{200}{9.81} \sin \theta \cos \theta$

$$R = \frac{100}{9.81} \sin 2\theta \qquad \textbf{(1)}$$

Require,

$$\frac{dR}{d\theta} = 0$$

$$\frac{100}{9.81} \cos 2\theta(2) = 0$$

$$\cos 2\theta = 0$$

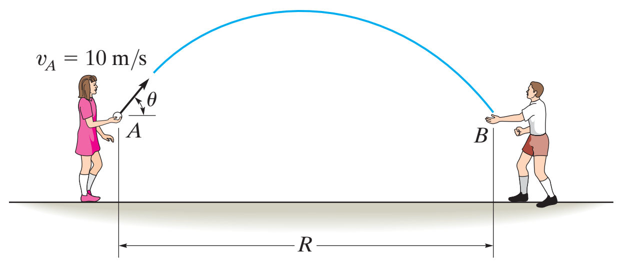
$\theta = 45°$ **Ans.**

$R = \dfrac{100}{9.81}(\sin 90°) = 10.2$ m **Ans.**

Ans:
$R_{max} = 10.2$ m
$\theta = 45°$

12–90.

Show that the girl at A can throw the ball to the boy at B by launching it at equal angles measured up or down from a 45° inclination. If $v_A = 10$ m/s, determine the range R if this value is 15°, i.e., $\theta_1 = 45° - 15° = 30°$ and $\theta_2 = 45° + 15° = 60°$. Assume the ball is caught at the same elevation from which it is thrown.



SOLUTION

$(\overset{+}{\rightarrow})\quad s = s_0 + v_0 t$

$$R = 0 + (10 \cos \theta)t$$

$(+\uparrow)\quad v = v_0 + a_c t$

$$-10 \sin \theta = 10 \sin \theta - 9.81t$$

$$t = \frac{20}{9.81} \sin \theta$$

Thus, $R = \dfrac{200}{9.81} \sin \theta \cos \theta$

$$R = \frac{100}{9.81} \sin 2\theta \qquad \textbf{(1)}$$

Since the function $y = \sin 2\theta$ is symmetric with respect to $\theta = 45°$ as indicated, Eq. (1) will be satisfied if $|\phi_1| = |\phi_2|$

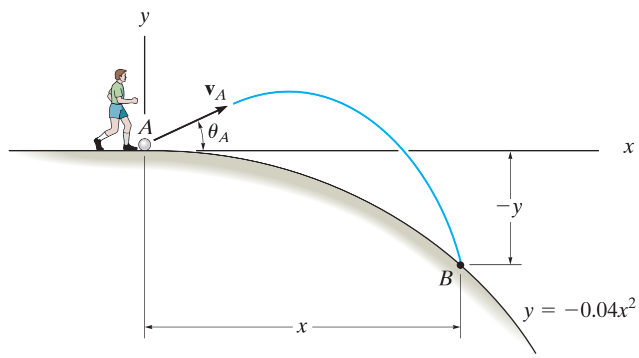
Choosing $\phi = 15°$ or $\theta_1 = 45° - 15° = 30°$ and $\theta_2 = 45° + 15° = 60°$, and substituting into Eq. (1) yields

$R = 8.83$ m **Ans.**

Ans:
$R = 8.83$ m

12–91.

The ball at A is kicked with a speed $v_A = 80$ ft/s and at an angle $\theta_A = 30°$. Determine the point $(x, -y)$ where it strikes the ground. Assume the ground has the shape of a parabola as shown.



SOLUTION

$(v_A)_x = 80 \cos 30° = 69.28$ ft/s

$(v_A)_y = 80 \sin 30° = 40$ ft/s

$(\overset{+}{\rightarrow})\ s = s_0 + v_0 t$

$x = 0 + 69.28t$ **(1)**

$(+\uparrow)\ s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$-y = 0 + 40t + \frac{1}{2}(-32.2)t^2$ **(2)**

$y = -0.04x^2$

From Eqs. (1) and (2):

$-y = 0.5774x - 0.003354x^2$

$0.04x^2 = 0.5774x - 0.003354x^2$

$0.04335x^2 = 0.5774x$

$x = 13.3$ ft **Ans.**

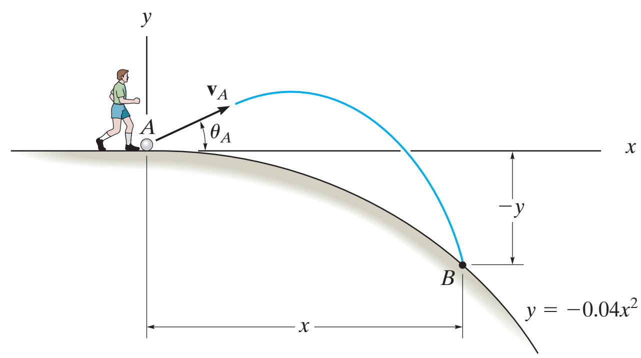
Thus

$y = -0.04\,(13.3)^2 = -7.09$ ft **Ans.**

Ans:
(13.3 ft, −7.09 ft)

***12–92.**

The ball at A is kicked such that $\theta_A = 30°$. If it strikes the ground at B having coordinates $x = 15$ ft, $y = -9$ ft, determine the speed at which it is kicked and the speed at which it strikes the ground.



SOLUTION

$(\overset{+}{\rightarrow})\quad s = s_0 + v_0 t$

$$15 = 0 + v_A \cos 30°\, t$$

$(+\uparrow)\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$-9 = 0 + v_A \sin 30°\, t + \frac{1}{2}(-32.2)t^2$$

$v_A = 16.5$ ft/s **Ans.**

$t = 1.047$ s

$(\overset{+}{\rightarrow})\quad (v_B)_x = 16.54 \cos 30° = 14.32$ ft/s

$(+\uparrow)\quad v = v_0 + a_c t$

$(v_B)_y = 16.54 \sin 30° + (-32.2)(1.047)$

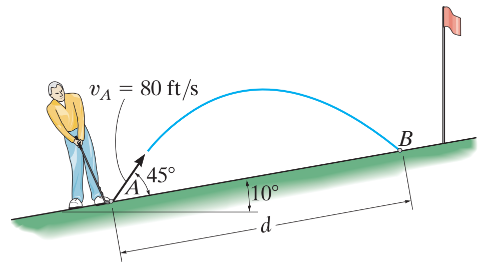
$= -25.45$ ft/s

$v_B = \sqrt{(14.32)^2 + (-25.45)^2} = 29.2$ ft/s **Ans.**

Ans:
$v_A = 16.5$ ft/s
$t = 1.047$ s
$v_B = 29.2$ ft/s

12–93.

A golf ball is struck with a velocity of 80 ft/s as shown. Determine the distance d to where it will land.



SOLUTION

$(\overset{+}{\rightarrow})\ s = s_0 + v_0 t$

$d \cos 10^\circ = 0 + 80 \cos 55^\circ\, t$

$(+\uparrow)\ s = s_0 + v_0\, t + \frac{1}{2} a_c t^2$

$d \sin 10^\circ = 0 + 80 \sin 55^\circ\, t - \frac{1}{2}(32.2)(t^2)$

Solving

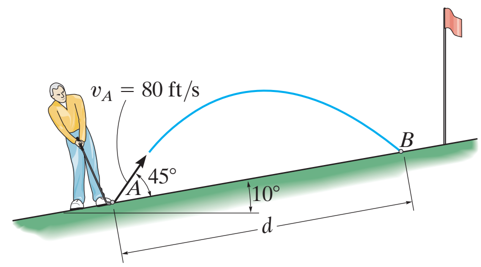
$t = 3.568$ s

$d = 166$ ft **Ans.**

Ans:
$d = 166$ ft

12–94.

A golf ball is struck with a velocity of 80 ft/s as shown. Determine the speed at which it strikes the ground at B and the time of flight from A to B.



SOLUTION

$(v_A)_x = 80 \cos 55° = 44.886$

$(v_A)_y = 80 \sin 55° = 65.532$

$(\overset{+}{\rightarrow})\quad s = s_0 + v_0 t$

$d \cos 10° = 0 + 45.886t$

$(+\uparrow)\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$d \sin 10° = 0 + 65.532\,(t) + \frac{1}{2}(-32.2)(t^2)$

$d = 166$ ft

$t = 3.568 = 3.57$ s **Ans.**

$(v_B)_x = (v_A)_x = 45.886$

$(+\uparrow)\quad v = v_0 + a_c t$

$(v_B)_y = 65.532 - 32.2(3.568)$

$(v_B)_y = -49.357$

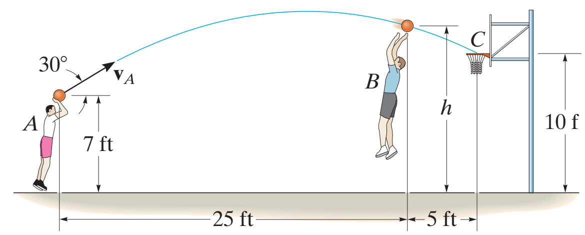
$v_B = \sqrt{(45.886)^2 + (-49.357)^2}$

$v_B = 67.4$ ft/s **Ans.**

Ans:
$t = 3.57$ s
$v_B = 67.4$ ft/s

12–95.

The basketball passed through the hoop even though it barely cleared the hands of the player B who attempted to block it. Neglecting the size of the ball, determine the magnitude v_A of its initial velocity and the height h of the ball when it passes over player B.



SOLUTION

$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$$30 = 0 + v_A \cos 30°\, t_{AC}$$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$10 = 7 + v_A \sin 30°\, t_{AC} - \frac{1}{2}(32.2)(t_{AC}^2)$$

Solving

$v_A = 36.73 = 36.7$ ft/s **Ans.**

$t_{AC} = 0.943$ s

$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$$25 = 0 + 36.73 \cos 30°\, t_{AB}$$

$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$h = 7 + 36.73 \sin 30°\, t_{AB} - \frac{1}{2}(32.2)(t_{AB}^2)$$

Solving

$t_{AB} = 0.786$ s

$h = 11.5$ ft **Ans.**

Ans:
$v_A = 36.7$ ft/s
$h = 11.5$ ft